EE 369C Lecture 8

Parallel MRI

Reconstruction of Images from Multiple Coils

Reconstruction of Undersampled Data

- Image Domain – SENSE (this time)
- Frequency Domain – SMASH/GRAPPA (next time)

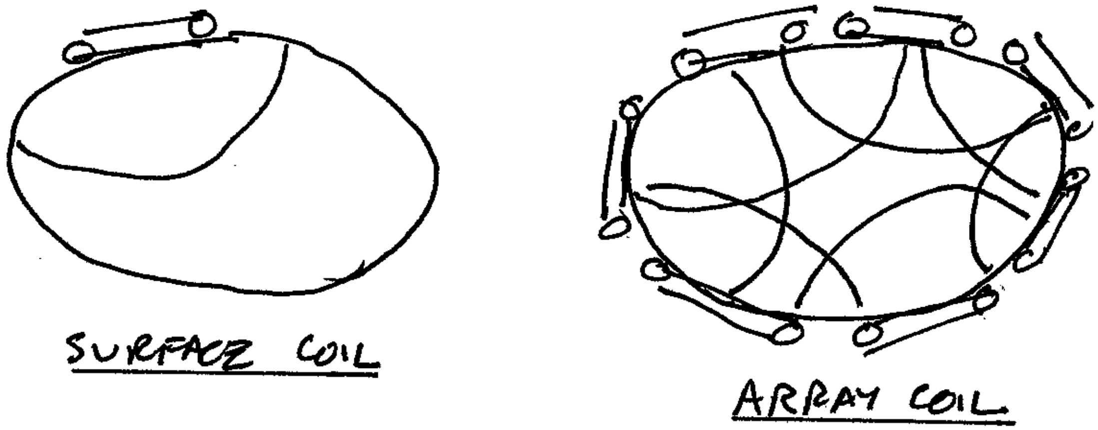
Reading

Larkman + Nunes Paper (online)

Section 13.3 in Bernstein

RECONSTRUCTION FROM MULTIPLE COILS

ROEMER, 1990



EACH COIL A COMPLETE IMAGE OF WHOLE FOV

EACH COIL HAS AN AMPLITUDE AND PHASE SENSITIVITY

$$C_\ell(\underline{x}) \qquad \ell = 1..L \quad \text{COILS}$$

COILS ARE COUPLED, SO NOISE IS CORRELATED

$$E[n_i n_j] = \psi \qquad \psi \text{ IS } L \times L$$

RECEIVED DATA FROM COIL ℓ

$$\underbrace{m_\ell(\underline{x})}_{\text{MEASURED IMAGES}} = \underbrace{C_\ell(\underline{x})}_{\text{COIL } \ell \text{ SENSITIVITY}} \underbrace{m(\underline{x})}_{\text{OBJECT}} + \underbrace{n_\ell(\underline{x})}_{\text{NOISE FROM COIL } \ell}$$

GIVEN $m_\ell(\underline{x})$, HOW DO WE RECONSTRUCT $m(\underline{x})$?

WE HAVE N MEASUREMENTS OF EACH VOXEL

$m_1(\underline{x})$ $m_2(\underline{x})$ $m_3(\underline{x})$ ... $m_L(\underline{x})$

FOR A PARTICULAR VOXEL $\underline{x}_p$

$$\begin{pmatrix} m_1(\underline{x}_p) \\ m_2(\underline{x}_p) \\ \vdots \\ m_L(\underline{x}_p) \end{pmatrix} = \begin{pmatrix} C_1(\underline{x}_p) \\ C_2(\underline{x}_p) \\ \vdots \\ C_L(\underline{x}_p) \end{pmatrix} m(\underline{x}_p) + \begin{pmatrix} n_1(\underline{x}_p) \\ n_2(\underline{x}_p) \\ \vdots \\ n_L(\underline{x}_p) \end{pmatrix}$$

OR

$$\underbrace{\underline{m}_s(\underline{x}_p)}_{\substack{L \times 1 \\ \text{MEASUREMENTS}}} = \underbrace{\underline{C}}_{\substack{L \times 1 \\ \text{SENSITIVITIES}}} \underbrace{m(\underline{x}_p)}_{\substack{1 \times 1 \\ \text{PIXEL}}} + \underbrace{\underline{n}}_{\substack{L \times 1 \\ \text{noise}}}$$

NOISE DISTRIBUTED AS

$$\underline{n} \sim N(0, \Psi)$$

MINIMUM VARIANCE ESTIMATE

$$\hat{m}(\underline{x}_p) = \underbrace{(C^* \Psi^{-1} C)^{-1}}_{1\times 1} \underbrace{C^* \Psi^{-1}}_{1\times L} \underbrace{\underline{m}_s(\underline{x}_p)}_{L\times 1}$$

COVARIANCE (VARIANCE)

$$\mathrm{Cov}(\hat{m}(\underline{x}_p)) = \underbrace{(C^* \Psi^{-1} C)}_{1\times 1}$$

SPECIAL CASE

IF

$$\Psi = \sigma^2 \underline{I}$$

THEN

$$\hat{m}(\underline{x}_p) = (C^*(\sigma^2 I)^{-1} C)^{-1} C^*(\sigma^2 I)^{-1} \underline{m}_s(\underline{x}_p)$$

$$= (C^* C)^{-1} C^* \underline{m}_s(\underline{x}_p)$$

$$= \frac{1}{\sum_\ell |c_\ell(x_p)|^2} \sum_\ell c_\ell^*(\underline{x}_p)\, m_{s,\ell}(\underline{x}_p)$$

WEIGHTED LINEAR COMBINATION

PHASE CORRECTED $(c_\ell^*(\underline{x}_p))$

WEIGHTING CORRECTED $\left(\frac{1}{\sum_\ell |c_\ell(x_p)|^2}\right)$

APPROXIMATE SOLUTION

OFTEN WE DON'T KNOW $C_l(\underline{x})$

HOWEVER

$$m_l(\underline{x}) = C_l(x)\, m(\underline{x})$$

SO $m_l(\underline{x})$ HAS $C_l(\underline{x})$ IMPLICITLY, PARTICULARLY
AMPLITUDE AND PHASE

APPROXIMATE SOLUTION

$$\hat{m}_{ss}(\underline{x}) = \sqrt{\sum_l m_l^*(\underline{x})\, m_l(\underline{x})}$$

THIS PHASE CORRECTS AND WEIGHTS THE MEASUREMENTS

SQUARE-ROOT APPROXIMATELY FIXES WEIGHTING
LEAVES SHADING

FOR SNR > 20, WITHIN 10% OF OPTIMAL
SOLUTION (ROEMER)

LONG THOUGHT TO BE ADEQUATE

ACCELERATED IMAGING WITH ARRAY COILS

PARALLEL IMAGING

BASIC IDEA

COIL ELEMENTS PROVIDE SOME LOCALIZATION

UNDERSAMPLE IN K-SPACE, PRODUCING ALIASING

SORT OUT IN RECONSTRUCTION

MANY APPROACHES

IMAGE DOMAIN - SENSE

FREQUENCY DOMAIN - SMASH, GRAPPA, SPIRIT

HYBRID - ARC, SPACE-RIP

WE WILL FOCUS ON TWO:

SENSE - OPTIMAL IF YOU KNOW COIL SENSITIVITIES

EASY TO VISUALIZE

EASY TO ANALYZE

GRAPPA -

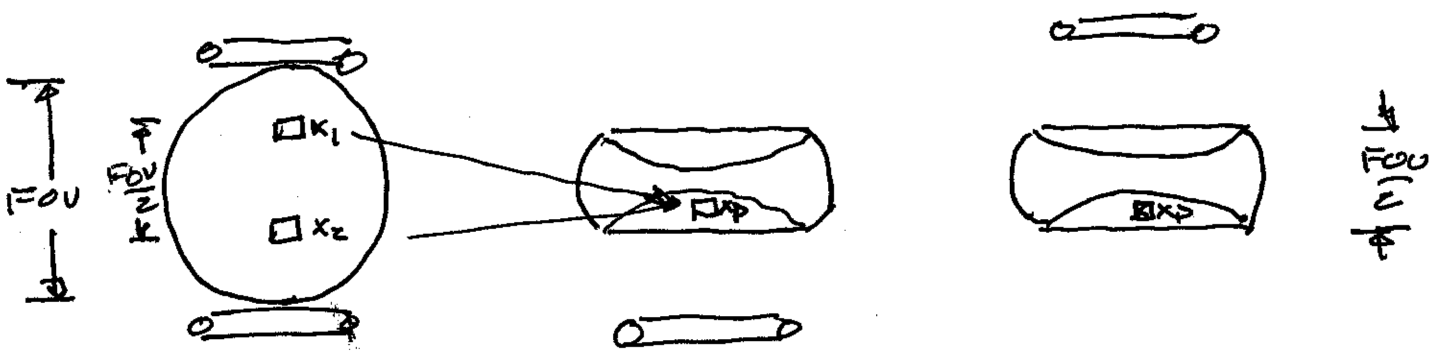
AUTOCALIBRATING

ROBUST

2DFT SENSE

L' RECEIVE COILS

UNDERSAMPLED IMAGES BY FACTOR OF 2
(R=2)



FULLY ENCODED — COIL 1 UNDERSAMPLED — COIL 2 UNDERSAMPLED

$\underline{x}_1$ AND $\underline{x}_2$ ALIAS TO $\underline{x}_p$

WE HAVE N MEASUREMENTS OF $\underline{x}_p$ (i.e. $\underline{x}_1 + \underline{x}_2$)
EACH WITH DIFFERENT SENSITIVITIES

$$\begin{array}{l} \text{COIL 1 –} \\ \\ \\ \\ \text{COIL N –} \end{array} \underbrace{\begin{pmatrix} m_1(\underline{x}_p) \\ m_2(\underline{x}_p) \\ \vdots \\ \vdots \\ m_L(\underline{x}_p) \end{pmatrix}}_{\text{ALIASED IMAGES}} = \underbrace{\begin{pmatrix} C_1(\underline{x}_1) & C_1(\underline{x}_2) \\ C_2(\underline{x}_1) & C_2(\underline{x}_2) \\ \vdots & \\ \vdots & \\ C_L(\underline{x}_1) & C_L(\underline{x}_2) \end{pmatrix}}_{\text{SENSITIVITY AT SOURCE VOXELS}} \underbrace{\begin{pmatrix} m(\underline{x}_1) \\ m(\underline{x}_2) \end{pmatrix}}_{\text{SOURCE VOXELS}} + \begin{pmatrix} n_1 \\ \\ \\ \\ n_L \end{pmatrix}$$

IN MATRIX FORM

$$\underbrace{\underline{m}_s(\underline{x}_p)}_{L\times 1} = \underbrace{\underset{\sim}{C}}_{L\times 2}\ \underbrace{\underset{\sim}{m}}_{2\times 1} + \underbrace{\underset{\sim}{n}}_{L\times 1}$$

SAME MINIMUM VARIANCE SOLUTION

$$\underbrace{\underset{\sim}{\hat{m}}}_{2\times 1} = \underbrace{\left(C^* \Psi^{-1} C\right)^{-1}}_{2\times 2} \underbrace{C^* \Psi^{-1}}_{2\times L} \underbrace{\underline{m}_s(\underline{x}_p)}_{L\times 1}$$

L ALIASED RECONSTRUCTIONS RESOLVE TWO IMAGE PIXELS

FOR AN NXN IMAGE, WE SOLVE

$\frac{N}{2} \times N$

2×2 INVERSE SYSTEMS

FOR HIGHER ACCELERATIONS, R, WE SOLVE

$\frac{N}{R} \times N$

R×R INVERSE SYSTEMS

MANY SMALL PROBLEMS, VERY FAST

HOW LARGE CAN R BE?

SNR COST

TWO SNR LOSS MECHANISMS

REDUCED SCAN TIME

CONDITION OF THE SENSE DECOMPOSITION

SNR LOSS

$$\frac{SNR_R}{SNR_F} = \frac{1}{g\sqrt{R}}$$

GEOMETRY FACTOR

REDUCED SCAN TIME

FULLY ENCODED COVARIANCE

$$\chi_F = \frac{1}{n_{K,F}}\left(C_F^{*}\Psi^{-1}C_F\right)^{-1}$$

VARIANCE OF ONE VOXEL, SCALAR

REDUCED ENCODING COVARIANCE

$$\chi_R = \frac{1}{n_{K,R}}\left(C_R^{*}\Psi^{-1}C_R\right)^{-1}$$

COVARIANCE OF R ALIASED VOXELS

FOR THE j^{th} ALIASED VOXEL

$$\frac{SNR_R}{SNR_F} = \frac{S/\sqrt{[\chi_R]_{j,j}}}{S/\sqrt{\chi_F}}$$

$$= \frac{1}{\sqrt{[\chi_R]_{j,j}/\chi_F}}$$

$$= \frac{1}{\sqrt{\frac{n_{k,F}}{n_{k,R}}}\sqrt{[(C_R^* \Psi^{-1} C_R)^{-1}]_{j,j}\,(C_F^* \Psi^{-1} C_F)}}$$

$$= \frac{1}{\sqrt{R}\;g}$$

WHERE

$$g = \sqrt{(C_F^* \Psi^{-1} C_F)\left[(C_R^* \Psi^{-1} C_R)^{-1}\right]_{j,j}}$$

GEOMETRY FACTOR, $g \geq 1$

OFTEN WRITTEN

$$g = \sqrt{[C^* \Psi^{-1} C]_{j,j}\,[(C^* \Psi^{-1} C)^{-1}]_{j,j}}$$

WHERE $C = C_R$, AND

$$[C^* \Psi^{-1} C]_{j,j} = (C_F^* \Psi^{-1} C)$$

FOR FULL ENCODING OF PIXEL j

GEOMETRY FACTOR IS CRITICAL

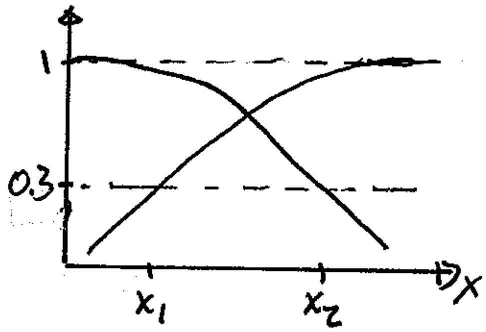
DEPENDS ON

- ACCELERATION
- SPATIAL POSITION
- ALIASING DIRECTION
- COIL GEOMETRY

MINIMIZING g DRIVES SYSTEM DESIGN

SENSE COILS ARE DIFFERENT FROM TRADITIONAL ARRAY COILS

EXAMPLE GOOD SENSE COIL



$C_1(x_1) = 1.0$
$C_1(x_2) = 0.3$
$C_2(x_1) = 0.3$
$C_2(x_2) = 1.0$

AT x_1

$$C_F = \begin{pmatrix} 1 \\ 0.3 \end{pmatrix} \qquad C_R = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$$

LET $\Psi = I$

$$g = \sqrt{(C_F^* C_F)\left[(C_R^* C_R)^{-1}\right]_{1,1}}$$

$$C_F^* C_F = (1 \quad 0.3)\begin{pmatrix} 1 \\ 0.3 \end{pmatrix} = 1.09$$

$$\chi_F = \frac{1}{n_{SF}}\,\frac{1}{1.09} = \frac{1}{n_{SF}}\,0.91$$

$$C_R^* C_R = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1.0 \end{pmatrix}\begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.09 & 0.6 \\ 0.6 & 1.09 \end{pmatrix}$$ SAME AS $C_F^* C_F$

$$(C_R^* C_R)^{-1} = \begin{pmatrix} 1.316 & -0.724 \\ -0.724 & 1.316 \end{pmatrix}$$

$$g = \sqrt{\underbrace{(1.09)}_{\text{MC LOSS}}\underbrace{(1.316)}_{\text{CONDITION LOSS}}} = 1.20$$

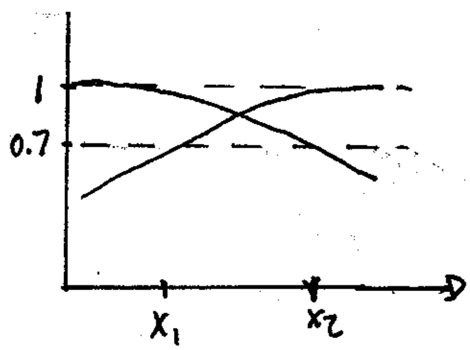
20% LOSS

L8

$$\frac{SNR_R}{SNR_F} = \frac{1}{\sqrt{2}\,(1.20)} = 0.59$$

40% SNR LOSS

EXAMPLE GOOD ARRAY COIL



$C_1(x_1) = 1$

$C_1(x_2) = 0.7$

$C_2(x_1) = 0.7$

$C_2(x_2) = 1$

AT X_1

$$C_F = \begin{pmatrix} 1 \\ 0.7 \end{pmatrix} \qquad C_R = \begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}$$

$$C_F^* C_F = 1.49$$

$$C_R^* C_R = \begin{pmatrix} 1.49 & 1.4 \\ 1.4 & 1.49 \end{pmatrix}$$

$$(C_R^* C_R)^{-1} = \begin{pmatrix} 5.72 & -5.38 \\ -5.38 & 5.72 \end{pmatrix}$$

$$g = \sqrt{(C_F^* C_F)\left((C_R^* C_R)^{-1}\right)_{1,1}}$$

$$= \sqrt{\underbrace{(1.49)}_{\text{MC LOSS}}\underbrace{(5.72)}_{\text{CONDITION}}} = 2.92$$

3X LOWER SNR

$$\frac{SNR_R}{SNR_F} = \frac{1}{\sqrt{2}\;2.92} = 0.24$$

4X LOWER SNR

SENSE COILS SHOULD BE DESIGNED FOR

- LOCAL SENSITIVITY
- AS ORTHOGONAL AS POSSIBLE
- OPTIMIZE SNR IN RECON

ARRAY COILS

- LARGER SENSITIVITIES
- APPROXIMATELY UNIFORM WITH SQUARE-ROOT OF SUM OF SQUARES RECON.