# EE 369C LECTURE 7

## OFF-RESONANCE CORRECTION

MULTIFREQUENCY RECONSTRUCTION

AUTOFOCUS RECONSTRUCTION

# MULTI-FREQUENCY RECONSTRUCTION

## DISCRETE FREQUENCY ALGORITHM

GOAL: TO RECONSTRUCT

$$s(t)e^{i(n\Delta\omega)t}$$

FOR $n = -L/2$ TO $L/2$

CHOOSE

$$\Delta\omega T = \pi/2$$

WHERE T IS THE READOUT DURATION. $\Delta\omega$ IS THEN 1/4 CYCLE OVER THE READOUT.

SIMPLE SOLUTION: DO L GRIDDING RECONSTRUCTIONS

L GRIDDING OPERATIONS

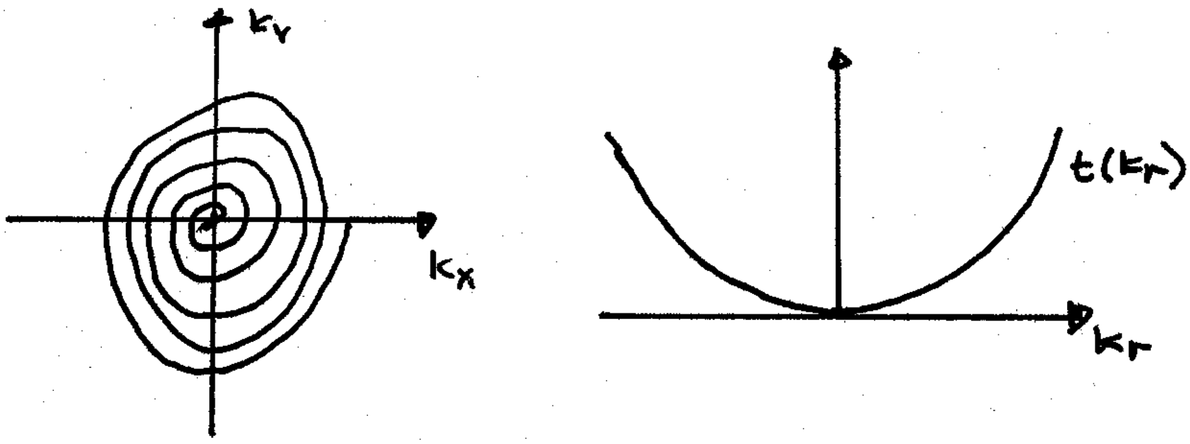
L 2D FFT'S

MANY FASTER ALTERNATIVES

# TIME MAPS

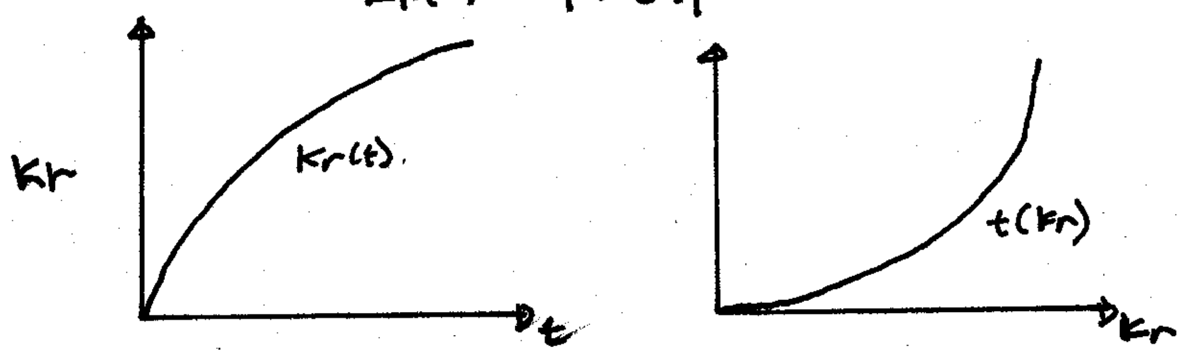
OFTEN WE CAN DEFINE A UNIQUE TIME FOR EACH K-SPACE POSITION

EXAMPLE: SPIRAL




WE FIND $t(k_r)$ BY INVERTING

$$k_r(t) = |k(t)|$$




DEFINE A PHASE FUNCTION

$$P(k_x, k_y) = e^{i \Delta\omega t(k_r)}$$

PHASE DUE TO ONE STEP IN $\Delta\omega$

IF $\hat{M}_0(k_x, k_y)$ IS THE RESULT OF GRIDDING $s(t)$

THEN

$$\hat{M}_1(k_x,k_y) = \hat{M}_0(k_x,k_y)\,P(k_x,k_y)$$
$$\hat{M}_{-1}(k_x,k_y) = \hat{M}_0(k_x,k_y)\,P^*(k_x,k_y)$$

AND

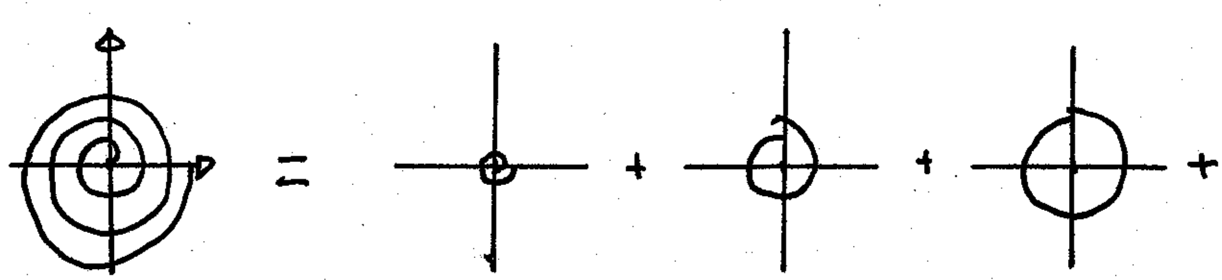
$$\hat{M}_\ell(k_x,k_y) = \hat{M}_{\ell-1}(k_x,k_y)\,P(k_x,k_y)$$
$$\hat{M}_{-\ell}(k_x,k_y) = \hat{M}_{-(\ell-1)}(k_x,k_y)\,P^*(k_x,k_y)$$

TOTAL OPERATIONS

1 GRIDDING OPERATION
L DIRECT MATRIX PRODUCTS
L 2D FFT'S

## DISCRETE TIME ALGORITHM

BREAK ACQUISITION INTO SHORT TIME SEGMENTS



EACH SEGMENT IS EFFECTIVELY A SINGLE TIME

RECONSTRUCT $m_\ell(x,y)$ FOR EACH SEGMENT

RECONSTRUCT AT $\omega$

$$m_\omega(x,y) = \sum_\ell m_\ell(x,y)\, e^{i\omega(\ell \Delta t)}$$

REQUIRES MORE SEGMENTS THAN DISCRETE FREQUENCY APPROACH.

TOTAL OPERATIONS

1 GRIDDING OPERATION ($L$ OF SIZE $1/L$)

$L$ 2D FFT'S

$L$ IMAGE COMBINATIONS

ADVANTAGE: DOESN'T REQUIRE A TIME MAP TO EXIST

BETTER INTERPOLATION IN FREQUENCY

BOTH CASES: NEAREST NEIGHBOR

SIGNIFICANT IMPROVEMENT WITH BETTER INTERPOLATION

FACTOR OF 2 FASTER FOR LARGE $L$ ($>10$)

# AUTOFOCUS RECONSTRUCTION

PROBLEMS WITH MAP BASED RECONSTRUCTION

MAPS TAKE ACQUISITION TIME

MAPS HAVE LIMITED RESOLUTION

MAPS ALSO SUFFER OFF-RESONANCE EFFECTS

MAPS FAIL AT BOUNDARIES, EDGES

AUTOFOCUS RECONSTRUCTION

DEFINE A FOCUS METRIC

OPTIMIZE ON A PIXEL BASIS

DOES BEST AT BOUNDARIES AND EDGES

## FOCUS METRICS

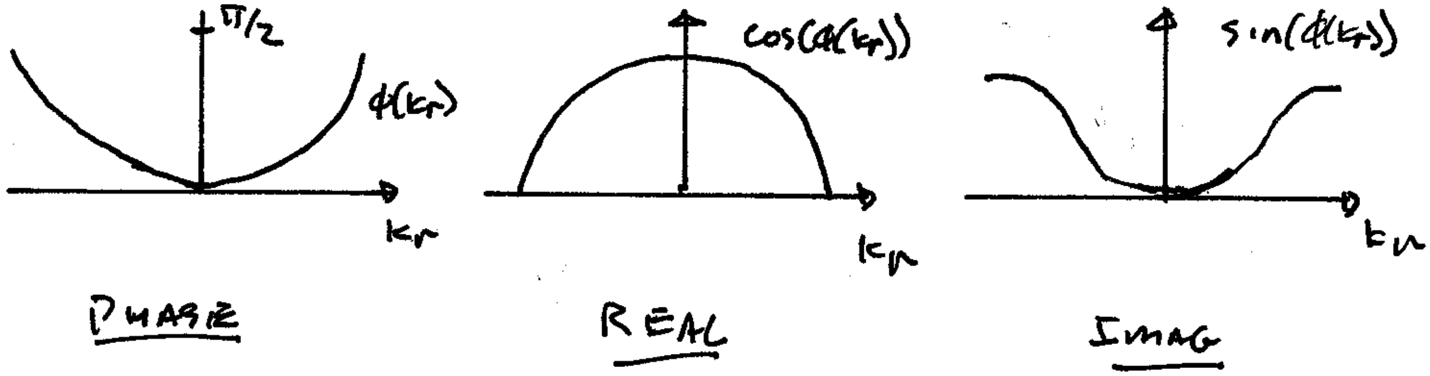
MANY POSSIBLE, DEPENDS ON PROBLEM

SIMPLEST: HIGH PASS FILTER

MAXIMIZE HIGH SPATIAL FREQUENCIES

FOR MR, WE ALSO HAVE PHASE WHICH IS USEFUL.

# SPIRAL, PR AUTOFOCUS




THE SIGNAL IS

$$M_w(k_x,k_y) = M(k_x,k_y)\,e^{i\phi(k_r)}$$

$$= \underbrace{M(k_x,k_y)\cos(\phi(k_r))}_{\text{LOW PASS}} + i\,\underbrace{M(k_x,k_y)\sin(\phi(k_r))}_{\text{HIGH PASS}}$$

IDEALLY, IF THE FREQUENCY WERE CORRECT, $\phi(k_r)=0$

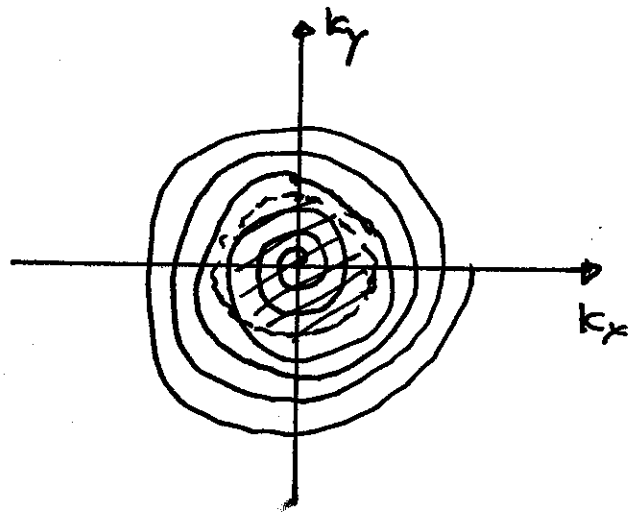
AND THERE WOULD BE <u>NO</u> IMAGINARY COMPONENT

# OFFSET PHASE CORRECTION

IF WE ARE GOING TO LOOK FOR THE IMAGINARY COMPONENT, WE NEED TO CORRECT FOR ALL OTHER SOURCES OF PHASE

COMPUTE A LOW RESOLUTION IMAGE USING A SMALL INITIAL PART OF DATA

USE THIS TO PHASE CORRECT MULTI-FREQUENCY RECONSTRUCTION




IF $m_{pc}(x,y)$ IS THE LOW RESOLUTION RECONSTRUCTION THEN

$$\underbrace{m_{\ell,pc}(x,y)}_{\text{PHASE CORRECTED MF RECON}} = \underbrace{m_{\ell}(x,y)}_{\text{MF RECON}} \cdot \underbrace{e^{-i \angle m_{pc}(x,y)}}_{\text{PHASE CORRECTION}}$$

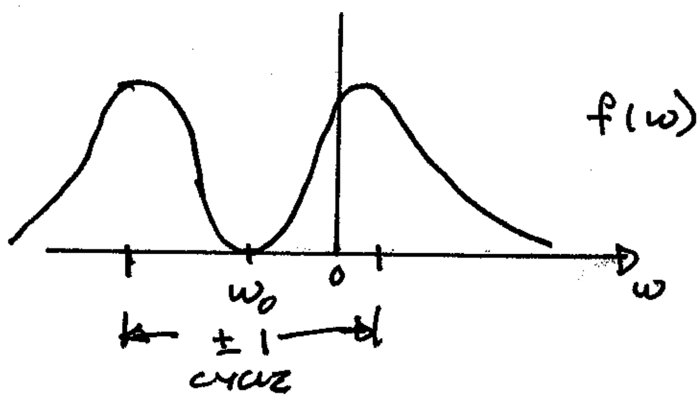
## AUTOFOCUS ALGORITHM

1) DO MULTI-FREQUENCY RECONSTRUCTION
2) PHASE CORRECT
3) AT EACH PIXEL, COMPUTE A FOCUS METRIC

$$f_\ell(x,y) = \sum_{W} \left| \mathrm{Imag}\{ m_{\ell,pc}(x,y) \} \right|^{\alpha}$$

W IS A SMALL $n \times n$ WINDOW CENTERED ON $x,y$

4) CHOOSE RECON WITH HIGHEST FOCUS METRIC




FOCUS METRIC AS A FUNCTION OF FREQUENCY

THE MINIMUM WILL BE FOUND IF THE RANGE IN FREQUENCIES IS LESS THAN ±1 CYCLE ($\pm\frac{1}{T}$ Hz)

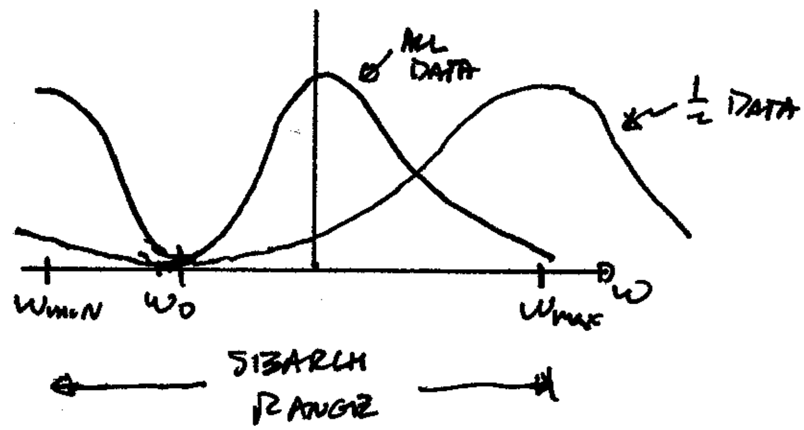
FURTHER AWAY, IT CAN GET LOST OUT ON THE TAILS, AND CHOOSE SOMETHING COMPLETELY WRONG!

## SOLUTION

DO AN INITIAL RECONSTRUCTION WITH THE 1/2 OF THE DATA.

THIS HAS TWICE THE UNAMBIGUOUS RANGE

DO THE FULL RECONSTRUCTION, BUT USE THE PREVIOUS RECONSTRUCTION TO LIMIT RANGE




ADD LEVELS FOR MORE BANDWIDTH

## QUESTIONS

WHAT IS THE RIGHT FOCUS METRIC?

HOW BIG SHOULD THE WINDOW BE?