EE 369C LECTURE 6

FINAL GRIDDING TOPICS

- INVERSE GRIDDING
- LEAST SQUARES PERSPECTIVE

OFF-RESONANCE CORRECTION

- EFFECTS OF OFF-RESONANCE ON NON-CARTESIAN MRI
- OFF-RESONANCE CORRECTION
  - MAP BASED RECONSTRUCTION
  - AUTOFOCUS RECONSTRUCTION

INVERSE GRIDDING

WE HAVE

CARTESIAN SAMPLED DATA IN IMAGE SPACE

WE WANT

NON-CARTESIAN SAMPLED DATA IN K-SPACE

NUFFT "PROBLEM 2"

WE'LL CALL IT "INVERSE GRIDDING"

BASIC IDEA

REVERSE GRIDDING OPERATIONS

1) PAD FOV WITH ZEROS

2) PRE-EMPHASIS

$$m_P(x,y) = \frac{m(x,y)}{c(x,y)} \text{Ш}\left(\frac{x}{\Delta x}, \frac{y}{\Delta y}\right)$$

3) IN K-SPACE

$$M_P(k_x,k_y) = M(k_x,k_y) * C^{-1}(k_x,k_y) * \text{Ш}(k_x\Delta x, k_y\Delta y)$$

WHERE

$$C^{-1}(k_x,k_y) \triangleq \mathcal{F}\left\{\frac{1}{c(x,y)}\right\}$$

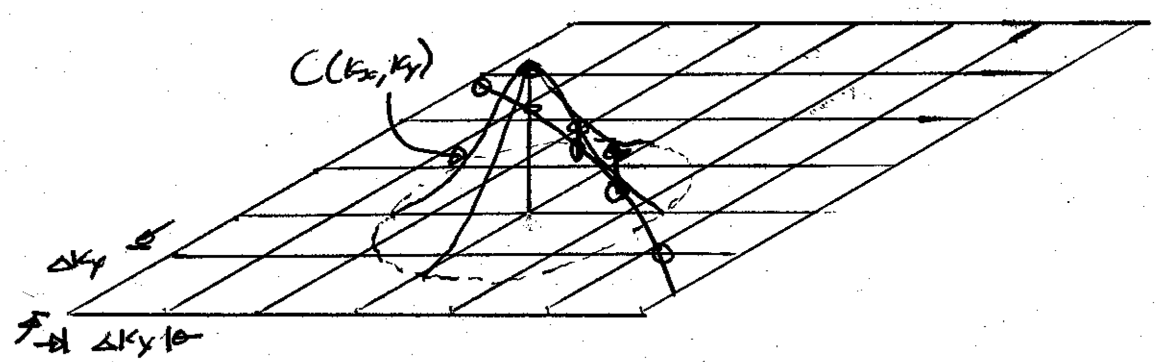
4) SAMPLE ON CARTESIAN GRID IN K-SPACE (THIS IS WHAT THE DFT DOES)

$$M_c(k_x,k_y) = \left(M(k_x,k_y) * C^{-1}(k_x,k_y) * Ш(k_x\Delta x, k_y\Delta y)\right) Ш\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)$$

5) CONVOLVE WITH THE KERNEL, RESAMPLE ON TRAJECTORY

$$\hat{m}(k_x,k_y) = \Big[\overbrace{\left(M(k_x,k_y) * C^{-1}(k_x,k_y) * Ш(k_x\Delta x, k_y\Delta y)\right) Ш\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)}^{M_D(k_x,k_y)}\Big] * C(k_x,k_y) \cdot S(k_x,k_y)$$

SAME AS GRIDDING, EXCEPT ADD CONTRIBUTION OF GRID POINT TO SAMPLES



<u>NO</u> DENSITY COMPENSATION

LEAST SQUARES PERSPECTIVE

WHAT OBJECT $\hat{m}(\underline{x})$ CORRESPONDS TO THE RECEIVED SIGNAL?

$$s(t_j) = \int_{\underline{x}} m(\underline{x})\, e^{-i2\pi \underline{k}(t_j)\cdot \underline{x}}\, d\underline{x}$$

$$\underbrace{s(t_j)}_{\underline{d}} \simeq \sum_{l} \underbrace{m(\underline{x}_l)}_{\underline{m}}\, \underbrace{e^{-i2\pi k(t_j)\cdot \underline{x}_l}}_{E}\, \underbrace{\Delta \underline{x}}$$

DATA — IMAGE — ENCODING MATRIX — VOXEL VOLUME

$$\underline{d} = E\, \underline{m}$$

WE ACQUIRE $\underline{d}$, AND KNOW E.

$$\underbrace{\begin{pmatrix} s(t_1) \\ s(t_2) \\ \vdots \\ s(t_j) \\ \vdots \\ s(t_n) \end{pmatrix}}_{\underline{d}} = \underbrace{\begin{pmatrix} \vdots & \vdots & & \vdots \\ e^{-i2\pi k(t_j)\cdot x_1} & e^{-i2\pi k(t_j)\cdot x_2} & \cdots & e^{-i2\pi k(t_j)\cdot x_m} \\ \vdots & \vdots & & \vdots \end{pmatrix}}_{E} \underbrace{\begin{pmatrix} m(\underline{x}_1) \\ \vdots \\ m(\underline{x}_l) \\ \vdots \\ m(\underline{x}_m) \end{pmatrix}}_{\underline{m}}$$

NORMALIZED SIGNAL FROM ONE VOXEL

ESTIMATE $\hat{\underline{m}}$ BY MINIMIZING

$$Q = (\underline{d} - E\hat{\underline{m}})^* (d - E\hat{m})$$

$$\hat{\underline{m}} = (E^* E)^{-1} E^* \underline{d}$$

TWO TERMS

$$E^* d = \sum_j e^{+i 2\pi \underline{k}(t_j) \cdot \underline{x}_k} d_j \qquad \boxed{\text{CONJUGATE PHASE RECON}}$$

$$E^* E = \sum_j e^{i 2\pi \underline{k}(t_j)(\underline{x}_k - \underline{x}_l)} \qquad \boxed{\text{IMPULSE RESPONSE}}$$

NOTE THAT IF $\underline{m}$ HAS A SINGLE NON-ZERO VOXEL AT $\underline{x}_l$

THEN

$$\underline{d} = e^{-i 2\pi k(t_j) \underline{x}_l}$$

AND THE FIRST TERM IS

$$E^* d = \sum_j e^{+i 2\pi \underline{k}(t_j) \underline{x}_k} e^{-i 2\pi \underline{k}(t_j) x_l}$$

$$= \sum_j e^{+i 2\pi \underline{k}(t_j)(\underline{x}_k - \underline{x}_l)}$$

THIS IS N (NUMBER OF SAMPLES) IF $\underline{x}_k = \underline{x}_l$.

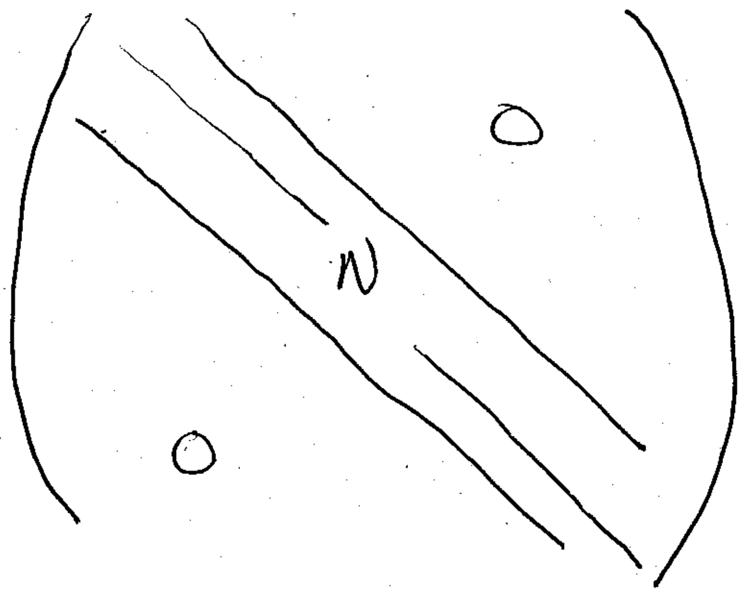
ESSENTIALLY THIS DEMODULATES THAT VOXEL, AND INTEGRATES.

THE SECOND TERM IS

$$\left(E^* E\right) = \sum_j e^{i 2\pi \underline{k}(t_j)(\underline{x}_k - \underline{x}_l)}$$

THIS IS A SQUARE MATRIX OF EVERY VOXEL DEMODULATED AT EVERY OTHER VOXEL.

$E^* E =$



IDEALLY THIS WOULD BE DIAGONAL

IN PRACTICE, IT GENERALLY WON'T BE

FOR 2DIFT, IT <u>IS</u> DIAGONAL

IF $E^* E$ IS NOT DIAGONAL, THE DEMODULATION (CONJUGATE PHASE) RECONSTRUCTION IS BLURRED

SOLUTION, AGAIN

$$\underline{\hat{m}} = \underbrace{(E^* E)^{-1}}_{\text{DECONVOLUTION}} \underbrace{E^* d}_{\text{BLURRED RECON}}$$

THE $E^* d$ IS A BLURRED RECONSTRUCTION DUE TO THE FACT THAT THE COLUMNS OF E ARE NOT ORTHOGONAL.

THE FACTOR $(E^* E)^{-1}$ IS A DECONVOLUTION OF THE IMPULSE RESPONSE

PROBLEM

- $E^* E$ IS $N^2 \times N^2$, AND OFTEN NOT SPARSE
  - EXPLICIT SOLUTION DIFFICULT
- ITERATIVE SOLUTIONS ARE EFFECTIVE
  - CONJUGATE GRADIENT ALGORITHMS
  - WE'LL SEE THIS AGAIN FOR NON-CARTESIAN PARALLEL IMAGE RECONSTRUCTION
- $E^* d$ IS A GRIDDING RECONSTRUCTION WITH NO DENSITY CORRECTION

WEIGHTED LEAST SQUARES

MINIMIZE

$$Q = (\underline{d} - E\underline{\hat{m}})^{*} W (\underline{d} - E\underline{\hat{m}})$$

$$\underline{\hat{m}} = (E^{*} W E)^{-1} E^{*} W \underline{d}$$

WHERE W IS A DIAGONAL WEIGHTING MATRIX.

WE WANT TO CHOOSE W SO THAT

$$(E^{*} W E) \cong I$$

THEN

$$\underline{\hat{m}} = \underbrace{(E^{*} W E)}_{I}{}^{-1} E^{*} W \underline{d}$$

$$\underline{\hat{m}} = E^{*} W \underline{d}$$

THIS IS A DENSITY CORRECTED CONJUGATE PHASE RECONSTRUCTION

THE W MATRIX THAT DOES THIS IS A DIAGONAL MATRIX WITH $1/\rho$ ON THE DIAGONAL

BRIEF INTRODUCTION TO OFF-RESONANCE CORRECTION

IN SPIN-WARP (2DFT) MRI, OFF-RESONANCE PRODUCES SPATIAL DISTORTION

$$\omega = \gamma G x \Leftrightarrow x$$

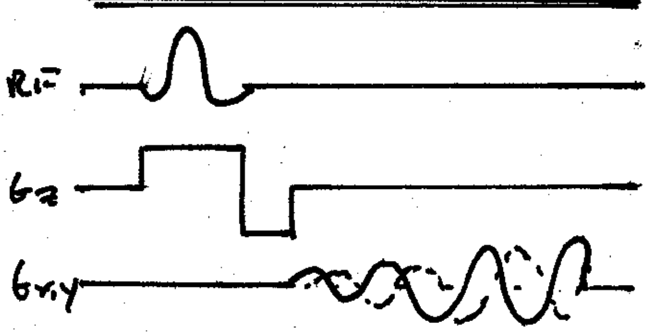
$$\Delta\omega \Leftrightarrow \Delta x$$

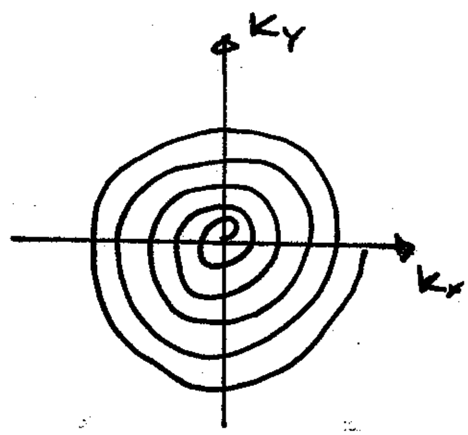
WE CAN'T TELL POSITION CHANGES FROM FREQUENCY CHANGES

PRODUCES BEAUTIFUL, SPATIALLY DISTORTED IMAGES

FOR NON-CARTESIAN MRI, PROBLEM IS MORE COMPLEX

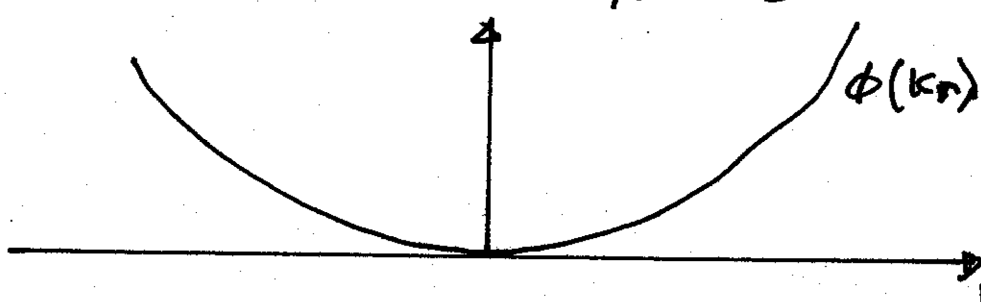
SPIRAL, RADIAL MRI



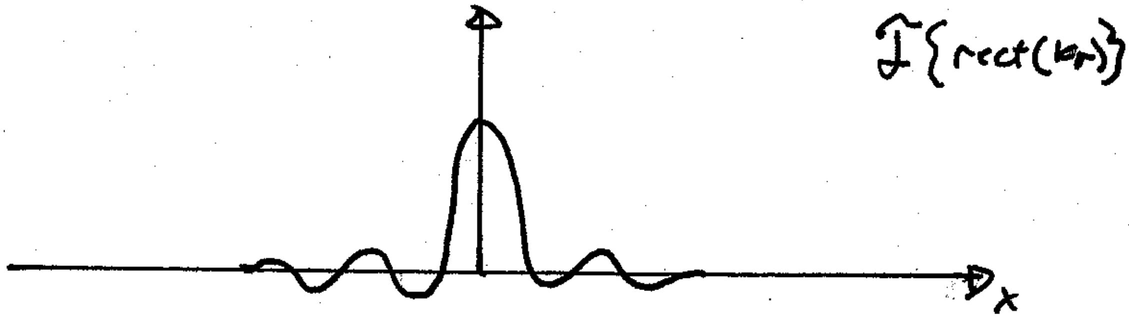


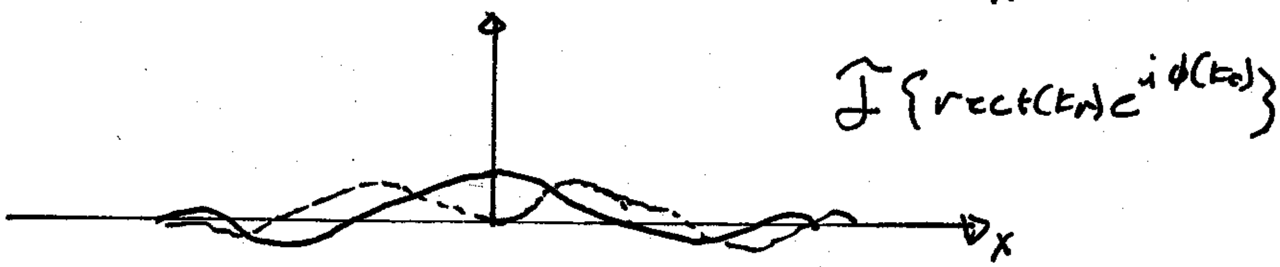
NO CONSTANT FREQUENCY AXIS

PHASE ACCRUES WITH TIME, RADIUS



RESULT IS BLURRING IN IMAGE SPACE





BROADER, LESS LOCALIZED IMPULSE RESPONSE

LOOKS EXACTLY LIKE SPHERICAL ABERRATION IN OPTICS

YOUR CAMERA FOCUSED AT THE WRONG DISTANCE

CAMERAS FOCUS AUTOMATICALLY

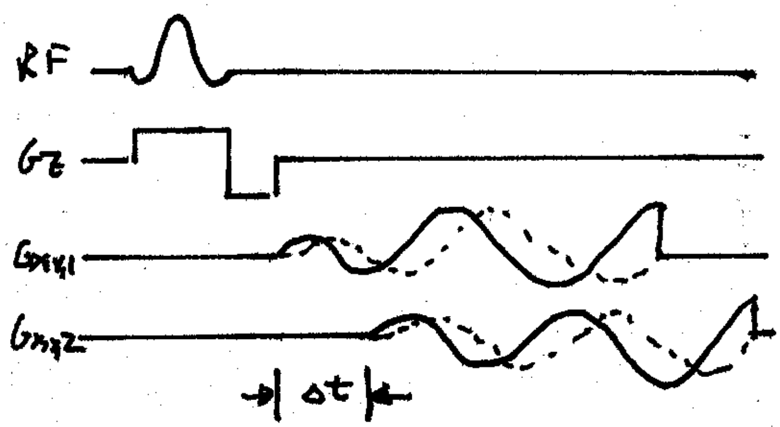
CAN WE DO THIS WITH MRI?

TWO APPROACHES:

1) MEASURE FIELD MAP, CORRECT
   a) B_0
   b) GRADIENT
   c) FIELD MAP
2) CHOOSE A FOCUSING METRIC, AND RECONSTRUCT AT A RANGE OF FREQUENCIES. CHOOSE BEST FREQUENCY FOR EACH VOXEL

FIELD MAPS

ACQUIRE IMAGES AT TWO ECHO TIMES



ACQ 1

ACQ 2

$$\Delta\phi(\underline{x}) = \angle\, m_{xy,2}(\underline{x})\, m^*_{xy,1}(\underline{x})$$

$$\omega(\underline{x}) = \frac{\Delta\phi(\underline{x})}{\Delta t}$$

$$f(x) = \frac{1}{2\pi}\,\omega(\underline{x}) \qquad \text{Hz}$$

DEMODULATION

SIGNAL EQUATION

$$s(t) = \int_{\underline{x}} m_{xy}(\underline{x})\, \underbrace{e^{-i\phi_c(\underline{x})}}_{\text{CONSTANT PHASE}}\, \underbrace{e^{-i\omega(\underline{x})t}}_{\text{OFF-RESONANCE}}\, \underbrace{e^{-i2\pi \underline{k}(t)\cdot\underline{x}}}_{\text{SPATIAL ENCODING}}\, d\underline{x}$$

$\phi_c(\underline{x})$ – CONSTANT, MANY SOURCES

$\omega(\underline{x})$ – LOCAL FREQUENCY

t – STARTS AT RF FOR GRE

AT SPIN ECHO FOR SPIN ECHOS

MULTI-FREQUENCY RECONSTRUCTION

ESTIMATE FREQUENCY FROM MAP

RECONSTRUCT AT L FREQUENCIES

FOR EACH PIXEL, PICK CLOSEST RECONSTRUCTION

QUESTIONS

HOW MANY FREQUENCIES

HOW TO COMPUTE IMAGES QUICKLY