LOW RANK FOR TIME SERIES RECONSTRUCTION

BASIC PROBLEM

SERIES OF IMAGES

SAME OBJECT

DIFFERENT CONTRAST -OR-

STATIC IMAGE + DYNAMIC COMPONENT

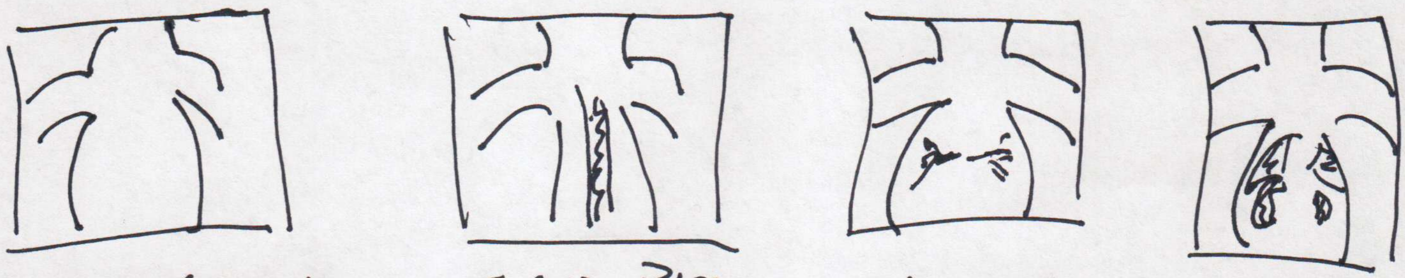
EXAMPLES

- DYNAMIC CONTRAST STUDIES
- T_1, T_2 PARAMETER ESTIMATION
- 4D FLOW
- IMAGE GUIDED INTERVENTIONS
  - THERMAL THERAPIES
  - INTERVENTIONAL DEVICES
- CARDIAC IMAGING

QUESTION

HOW DO YOU EXPLOIT TEMPORAL REDUNDANCY FOR FASTER IMAGING?

EXAMPLE DYNAMIC CONTRAST IMAGING

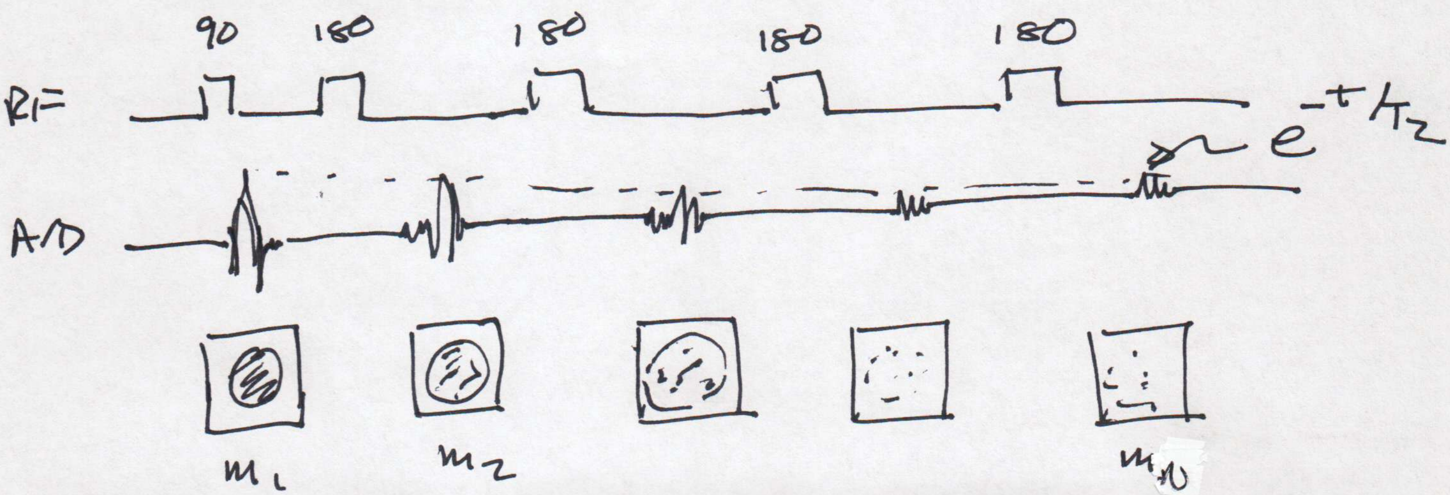


PRE-CONTRAST — FIRST PASS — VASCULAR EARLY — LATE ORGANS

SAME OBJECT, DIFFERENT CONTRAST

EXAMPLE T_2 MAPPING

PULSE SEQUENCE



AGAIN, SAME OBJECT, DIFFERENT CONTRAST, SIGNAL LEVELS

WE HAVE A STACK OF N IMAGES

$$m = \{m_i\} \qquad (n_x, n_y, N)$$

WE'D LIKE TO RECONSTRUCT THESE ALL TOGETHER

LET C BE AN OPERATOR THAT VECTORIZES EACH IMAGE

$$Cm = \begin{pmatrix} \vdots & \vdots & & \vdots \\ m_1 & m_2 & \cdots & m_N \\ \vdots & \vdots & & \vdots \end{pmatrix} \qquad (n_x n_y, N)$$

THIS IS THE <u>CASORATI</u> MATRIX

WHAT WE WANT IS THE <u>SIMPLEST</u> REPRESENTATION OF THIS DATA

IN PRACTICE, THIS MEANS WE TAKE THE MINIMIZATION

$$\text{minimize}_m \; \|Cm\|_*$$

WHERE $\|C_m\|_*$ IS THE NUCLEAR NORM OF C_m, WHICH IS THE SUM OF THE SINGULAR VALUES OF C_m.

WHY NUCLEAR NORM?

SINGULAR VALUES ARE NON-NEGATIVE

SINGULAR VALUES ARE THE SQUARE ROOT OF THE EIGENVALUES OF C^*C OR CC^*

EFFECTIVELY L1 NORM OF SVD

PROMOTES SPARSITY

SOLVED WITH FISTA (ITERATIVE THRESHOLDED SHRINKAGE)

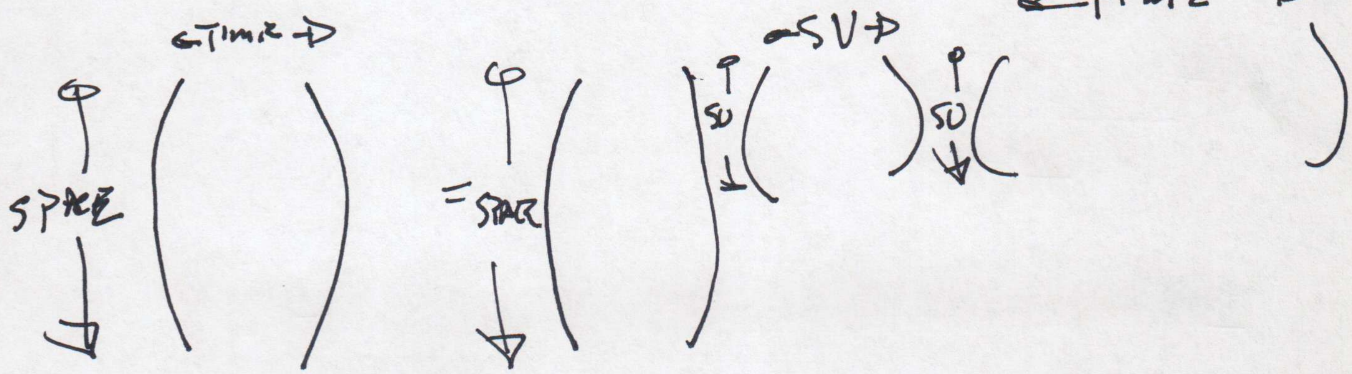
WHAT DOES THIS DO IN PRACTICE?

LEFT SINGULAR VECTORS

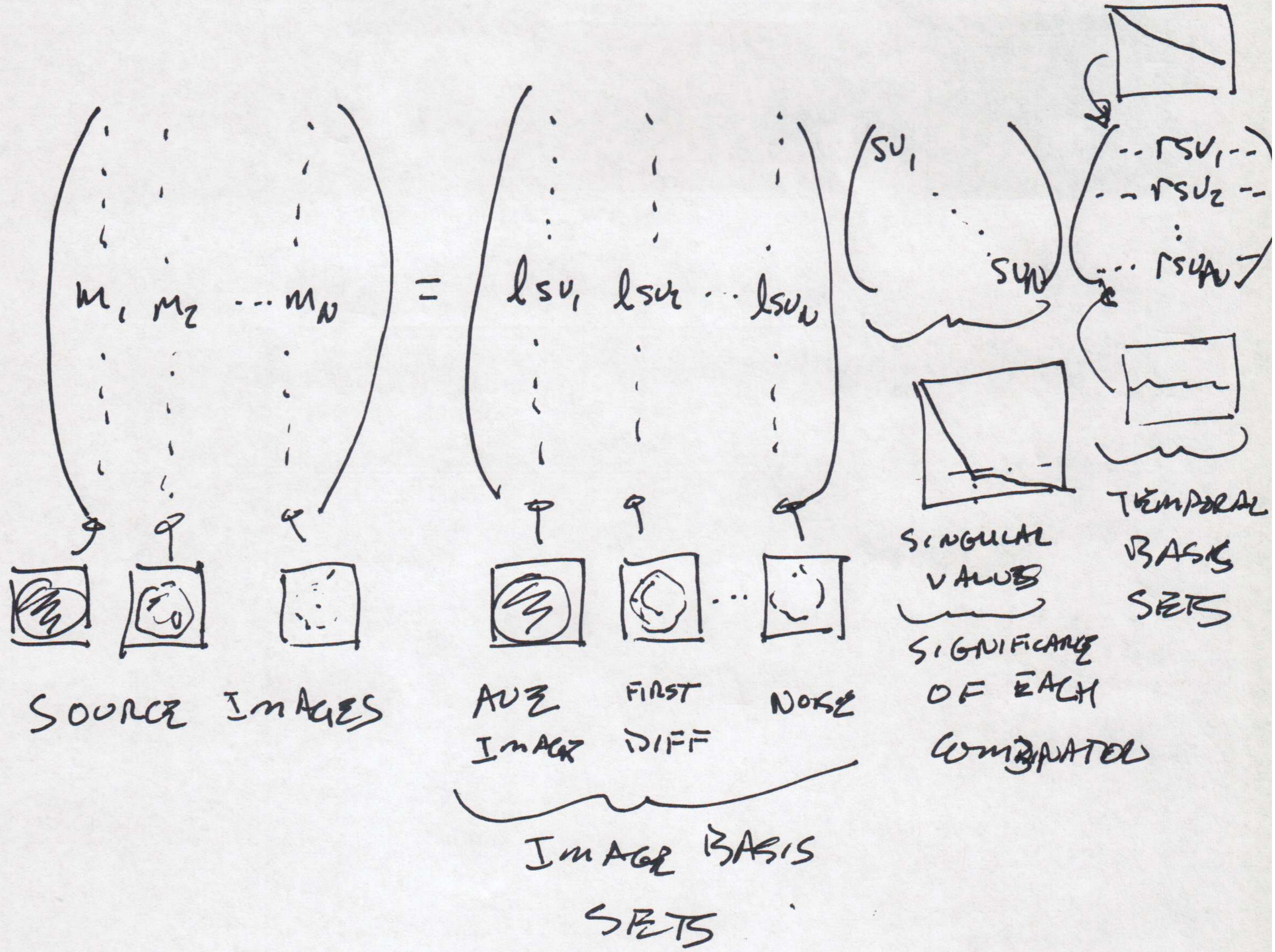
SINGULAR VALUES

RIGHT SINGULAR VECTORS

$$C_m = U \Sigma V$$



SPECIFICALLY FOR T2 MAPPING



IN PRACTICE, THE SINGULAR VALUES WILL FALL OFF RAPIDLY

THIS CAN BE EXPLOITED FOR

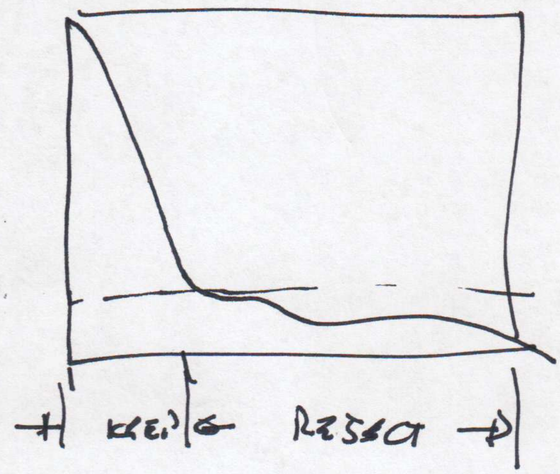
DENOISING

COMPRESSED SENSING

DENOISING

ASSUME M IS FULLY SAMPLED

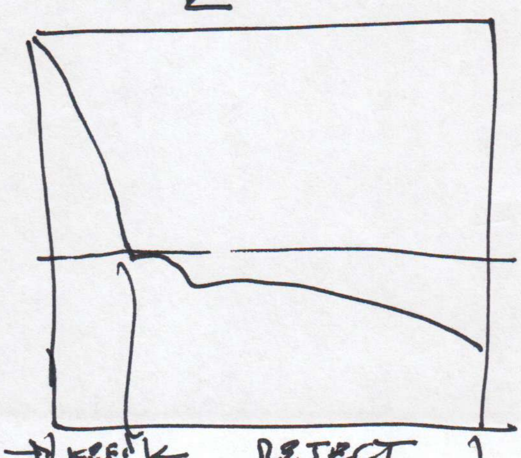
IF WE ZERO OUT SINGULAR VALUES BELOW A THRESHOLD WE CAN REJECT NOISE AND IMPROVE SNR



COMPRESSED SENSING

ASSUME EACH IMAGE HAS A UNIQUE RANDOM UNDERSAMPLING PATTERN

RANDOM SAMPLING ⇒ INCOHERENT ARTIFACTS
THESE WILL BE INDEPENDENT FROM IMAGE TO IMAGE



UNDERSAMPLING LOOKS LIKE HIGHER NOISE BACKGROUND

HIGHER THRESHOLD

COMPRESSED SENSING RECONSTRUCTION (7)

MINIMIZE

$$\min_m \left\{ \underbrace{\| Cm \|_*}_{\text{LOW RANK}} + \overbrace{\lambda_m}^{\text{TRADE OFF BETWEEN LR+CS}} \underbrace{\| DFm - y \|_2}_{\text{DATA CONSISTANCY}} \right\}$$

WHERE F IS FOURIER TRANSFORM OPERATOR, AND D IS SUBSAMPLING

WE DO THIS WITH THE SAME ITERATIVE COMPRESSED SENSING RECONSTRUCTION

ITERATE OVER

REPLACE WAVELET TRANSFORM

1) COMPUTE SVD OF CASORATI MATRIX, AND ZERO OUT LOW VALUES, SHRINK THE REST (FISTA)

2) ENFORCE DATA CONSISTANCY IN SPATIAL FREQUENCY

GLOBAL LOW RANK VS LOCAL LOW RANK

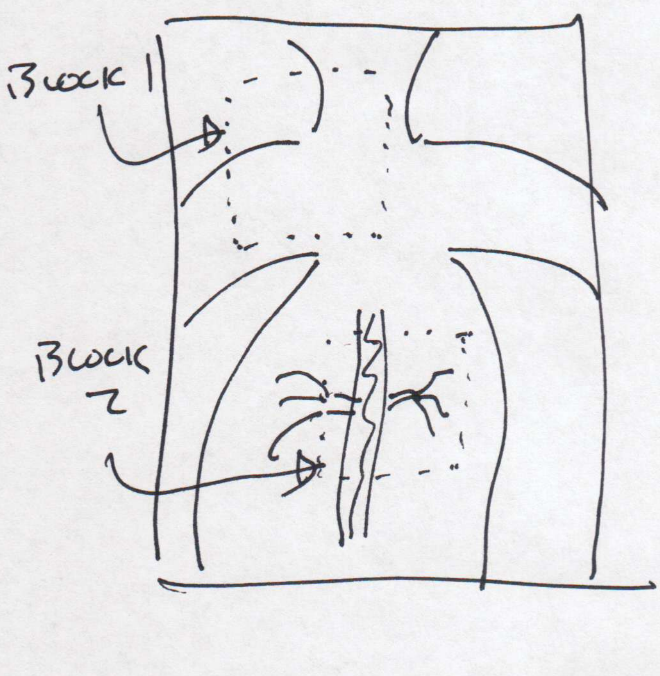
SO FAR, WE HAVE INCLUDED THE WHOLE IMAGE IN THE LOW RANK DECOMPOSITION

GLOBAL LOW RANK (GLR)

THIS WORKS, BUT WE CAN DO BETTER BY BREAKING THE IMAGE INTO BLOCKS

BETTER REPRESENTS LOCAL VARIATIONS

LOCAL LOW RANK (LLR)



BLOCK 1: NOTHING CHANGES WE CAN USE THE ENTIRE TIME SERIES

BLOCK 2: WE HAVE SEVERAL TIME COURSES

1) BACKGROUND
2) FIRST PASS
3) DELAYED ENHANCEMENT

DIFFERENT REPRESENTATIONS AS A FUNCTION OF POSITION

EXTENTION TO PARALLEL IMAGING

IN PRACTICE WE HAVE MULTIPLE RECEIVE CHANNELS

THIS ALLOWS GREATER SUBSAMPLING

THE SOURCE IMAGES ARE NOW FROM EACH CHANNEL, AND THE m_i ARE (n_x, n_y, n_c)

THE CASORATI OPERATOR VECTORIZES ALL OF THE CHANNELS

$$C_m = \begin{pmatrix} | & | & & | \\ m_{11} & m_{21} & & m_{N1} \\ | & | & & | \\ m_{12} & m_{22} & & m_{N2} \\ | & | & & | \\ m_{13} & m_{23} & \cdots & m_{N3} \\ | & | & & | \\ m_{1M} & m_{2M} & & m_{NM} \end{pmatrix}$$

PARALLEL IMAGING ALLOWS MUCH GREATER SUBSAMPLING

COMPRESSED SENSING, PARALLEL IMAGING

LOCALLY LOW RANK RECONSTRUCTION

THIS IS SOLVED ITERATIVELY BY CYCLING BETWEEN

1) PI RECONSTRUCTION

2) LLR EXPANSION, THRESHOLDING

3) DATA CONSISTANCY

THIS WORKS WELL WHEN THE UNDERLYING OBJECT IS THE SAME, AND ONLY THE CONTRAST VARIES.

LOW RANK + SPARSE (LR+CS)

WE FREQUENTLY HAVE A

RELATIVELY STATIC BACKGROUND COMPONENT

DYNAMIC FORGROUND COMPONENT

EXAMPLES:

CARDIAC IMAGING

CHEST WALL IS CONSTANT, HEART IS MOVING

INTERVENTIONAL IMAGING

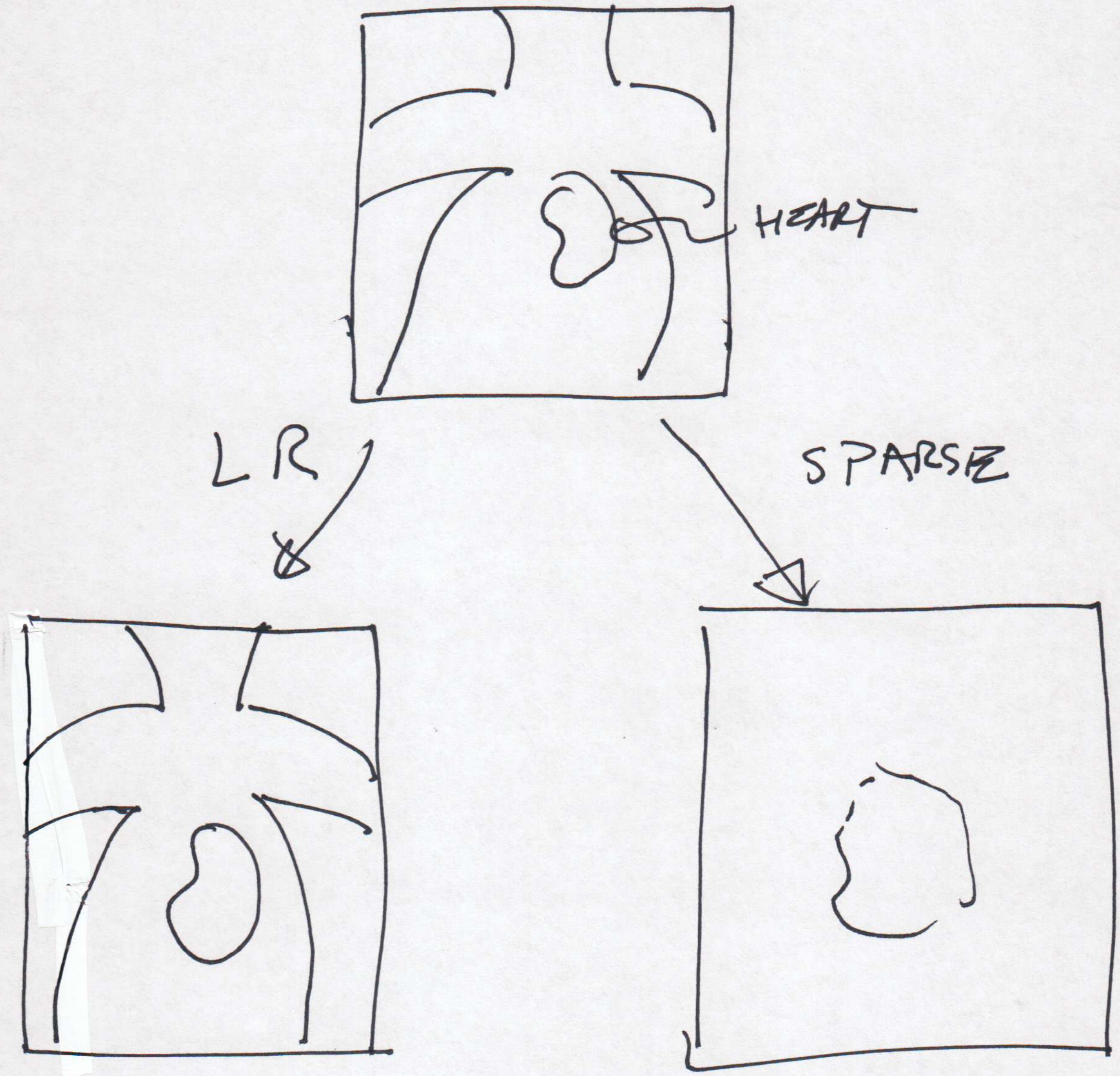
ANATOMY IS CONSTANT, DEVICE IS MOVING

HOW DO WE RECONSTRUCT THESE TIME SERIES?

SOLUTION: REPRESENT THE IMAGE SEQUENCE AS A COMBINATION OF A LOW RANK COMPONENT (BACKGROUND) AND A SPARSE COMPONENT (DYNAMIC).

AFTER ELIMINATING THE BACKGROOD LOW RANK COMPONENT, THE DYNAMIC DATA IS VERY SPARSE!

EXAMPLE CARDIAC IMAGING



AVERAGE IMAGE

CHANGING POSITION

ELIMINATING LR COMPONENT MAKES
RESIDUAL VERY SPARSE

SUPPORTS A LARGE ACCELERATION FACTOR

SOLUTION IS TO MINIMIZE

$$\min_m \left\{ \underbrace{\|DFm - y\|_2}_{\text{DATA CONSISTANCY}} + \lambda_L \underbrace{\|Cm\|_*}_{\text{LOW RANK}} + \lambda_S \underbrace{\|Tm\|}_{\text{SPARCITY}} \right\}$$

THE VALUES λ_S AND λ_L TRADE OFF SPARCITY, LOW RANK AND DATA CONSISTANCY

RECONSTRUCTION ALGORITHM CYCLES THROUGH THE VARIOS CONSTRAINTS

AGAIN, THIS EXTENDS TO PARALLEL IMAGING.

CONCLUSION

IF WE HAVE A TIME SERIES OF IMAGES FROM THE SAME OBJECT, WE CAN GREATLY ACCELERATE RECONSTRUCTION

THE LOW RANK CONSTRAINT ADDS ANOTHER ACCERATION FACTOR

$$R_T = R_P \cdot R_C \cdot R_{LR}$$

WE'VE SEEN THAT R_P AND R_C ARE ON THE ORDER OF 2-4 FOR STANDARD CLINICAL IMAGES

THE ADDITION OF LOW RANK BRINGS THE TOTAL ACCERATION TO 15-20!

THIS IS ENABLING FOR STUDIES LIKE 4D FLOW, THAT WOULD OTHERWISE TAKE AN HOUR OR MORE.