LECTURE 16

PET RECONSTRUCTION

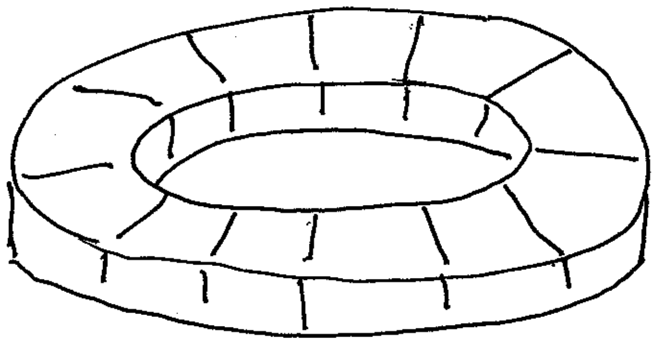
LAST TIME

- PHYSICAL MODEL
- FILTERED BACKPROJECTION

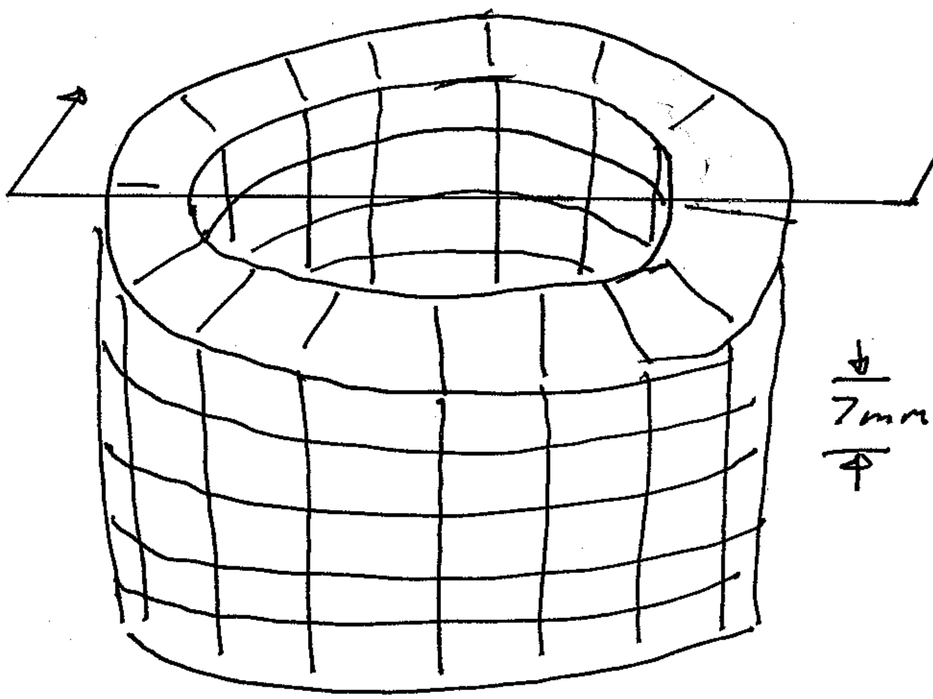
THIS TIME

- 3D PET
- ESTIMATION BASED RECONSTRUCTION

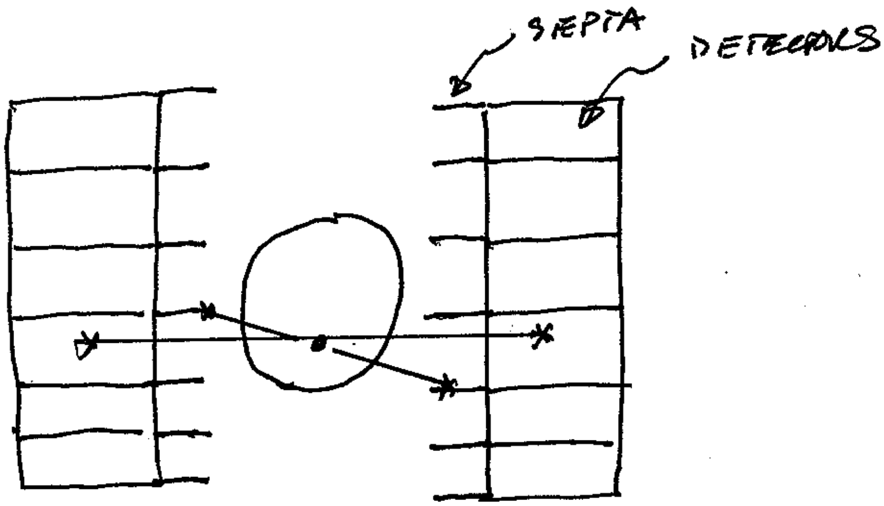
SO FAR WE HAVE ASSUMED WE HAVE A SINGLE RING OF DETECTORS



MOST SYSTEMS HAVE MULTIPLE RINGS



IN CROSS SECTION



RETRACTABLE SEPTA LIMIT DETECTED EVENTS TO PLANE

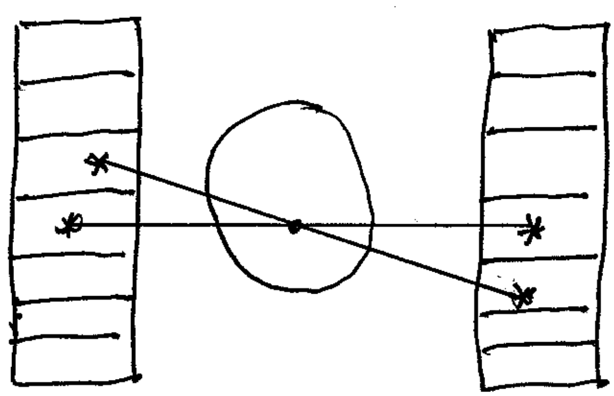
MULTIPLE SINGLE RING SYSTEMS

LESS EFFICIENT, MANY PHOTONS LOST

EASIER RECONSTRUCTION

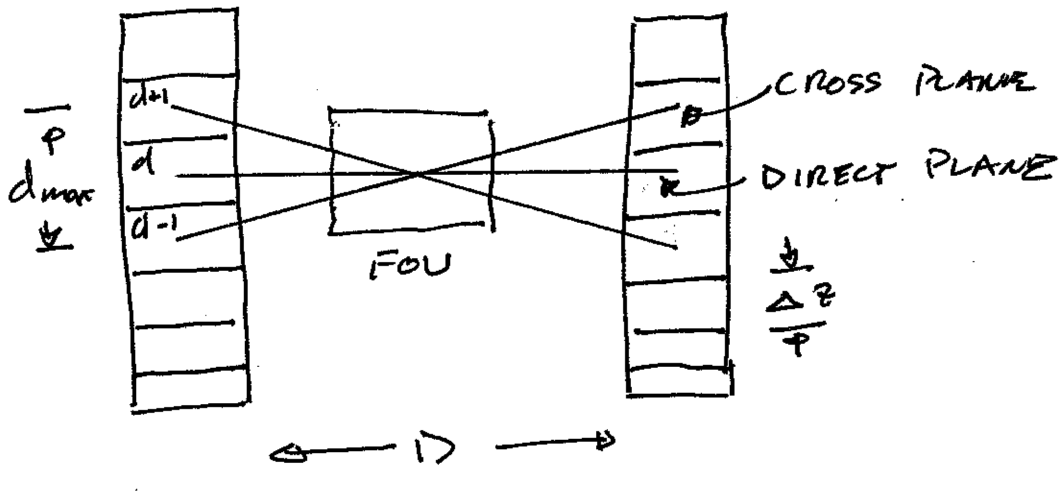
FEWER RANDOMS, LESS SCATTER

WITH SEPTA RETRACTED, FULL 3D SYSTEM



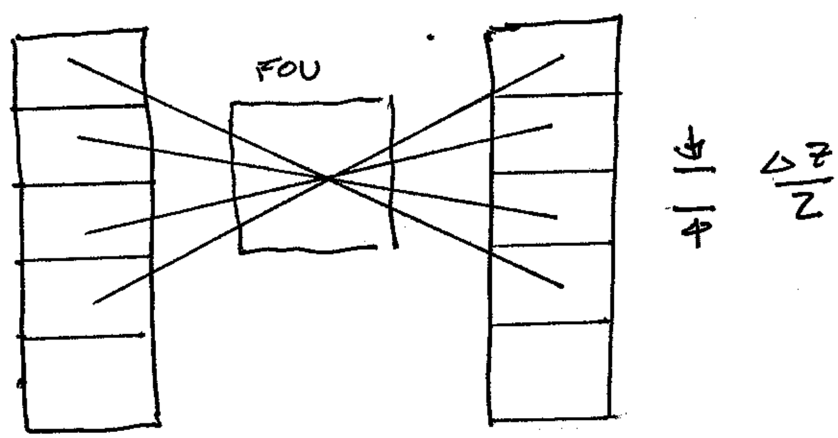
VOLUMETRIC DATA
MUCH MORE EFFICIENT
MORE DIFFICULT RECONSTRUCTION

SIMPLE 3D EXTENSION



IF FOV < D, THEN $d+1 - d-1$ LOR IS APPROXIMATELY COPLANAR WITH $d-d$ LOR

IF d_{max} IS EVEN

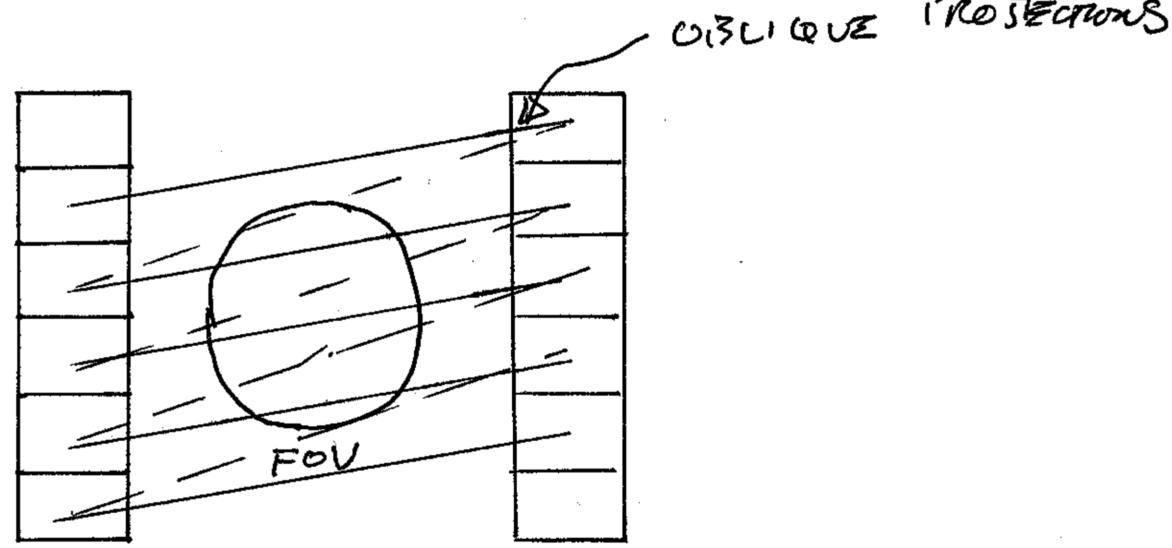


WE RECONSTRUCT PLANE AT $\Delta z/2$

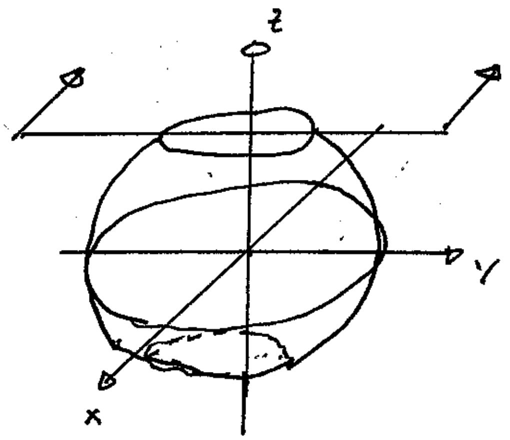
SENSITIVITY INCREASES $\sim d_{max}$

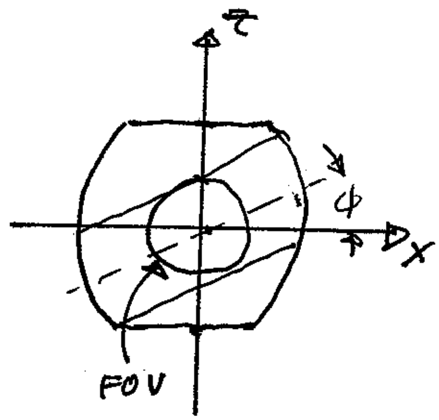
MORE PHOTONS CAPTURED

FULL 3D RECONSTRUCTION



PROJECTIONS COVER A TRUNCATED SPHERE



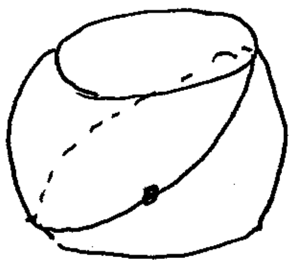


DATA SET IS OVERCOMPLETE

AXIAL PROJECTIONS ARE SUFFICIENT

OBLIQUE PROJECTIONS

ANY GREAT CIRCLE IS A COMPLETE DATA SET



OBLIQUE

AXIAL

RECONSTRUCT BY BACKPROJECTING EACH PARALLEL-
PROJECTION

DATA APPEARS IN MORE THAN ONE GREAT CIRCLE

DENSITY CORRECTION PROBLEM

INCORPORATE INTO RHO FILTER, NOW ELEVATION
ANGLE DEPENDENT

ESTIMATION BASED RECONSTRUCTION

SYSTEM MODEL

PROJECTION DATA

$$\underbrace{\underline{Y}}_{\text{PROJECTION DATA}} = \overset{\text{SYSTEM MATRIX}}{A}\,\underbrace{\underline{\lambda}}_{\text{VOXEL VALUES}} + \underbrace{\underline{S}}_{\text{SCATTER}} + \overset{\text{RANDOM}}{\underline{r}}$$

INCLUDE $\underline{S}$, AND $\underline{r}$ INTO A

$$\underline{Y} = A\underline{\lambda}$$

A IS THE SYSTEM MATRIX. IT IS BASICALLY A HIGH FIDELITY PET SIMULATOR.

THE $\underline{Y}$ AND $\underline{\lambda}$ ARE VECTORS OF INDEXED PROJECTION DATA y_i AND VOXEL VALUES λ_j

$$y_i = \sum_j^P a_{ij}\lambda_j$$

PROJECTION OF P VOXELS (SECOND INDEX)

ALSO

$$\hat{\underline{\lambda}} = A^T \underline{y}$$

$$\hat{\lambda}_j = \sum_i^N a_{ij} y_i$$

BACKPROJECTION OF N LOR'S (FIRST INDEX)

NOT FILTERED

GOAL

WE WANT TO FIND $\underline{\lambda}$ SUCH THAT

$$Pr(\underline{\lambda}|\underline{Y})$$

IS MAXIMIZED. MAXIMUM A POSTERIORI (MAP). BY BAYES RULE

$$Pr(\underline{\lambda}|\underline{Y}) = \frac{Pr(\underline{Y}|\underline{\lambda})\,Pr(\underline{\lambda})}{Pr(\underline{Y})}$$

TAKING LOG

$$Q(\underline{\lambda},\underline{Y}) = \underbrace{\log(Pr(\underline{Y}|\underline{\lambda}))}_{\text{LIKELYHOOD}} + \underbrace{\log(Pr(\underline{\lambda}))}_{\text{PRIOR INFORMATION (SUPPORT, SMOOTHNESS)}} - \underbrace{\cancel{\log(Pr(\underline{Y}))}}_{\text{MEASUREMENT CONSTANT}}$$

WE WILL LOOK AT MAXIMIZING LIKELYHOOD (ML).

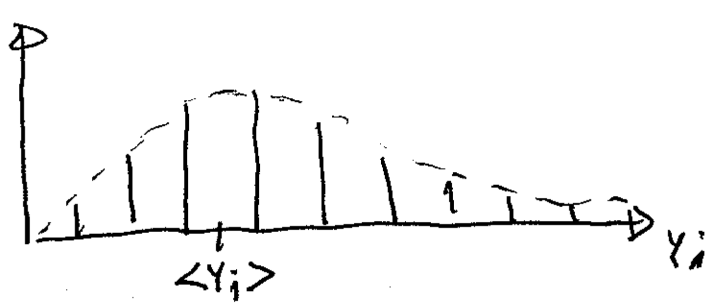
PET IS A COUNTING PROCESS, POISSON STATISTICS

$$\langle Y_i \rangle = \sum_{1}^{P} a_{i,j}\lambda_j$$

PROJECTION OF P VOXELS

$$Pr(\underline{Y}|\underline{\lambda}) = \prod_{i=1}^{N} \underbrace{e^{-\langle Y_i \rangle}\left(\frac{\langle Y_i \rangle^{Y_i}}{Y_i!}\right)}_{\text{POISSON DISTRIBUTION}}$$

HOW PROBABLE IS Y_i?



SUBSTITUTE INTO $Q(\underline{\lambda}, \underline{Y})$ (9)

$$Q(\underline{\lambda},\underline{Y}) = \log\left[\prod_{i=1}^{N} e^{-\langle Y_i \rangle} \frac{\langle Y_i \rangle^{Y_i}}{Y_i!}\right]$$

$$= \sum_{i=1}^{N} (-\langle Y_i \rangle) + Y_i \log \langle Y_i \rangle - Y_i!$$

$$= \sum_{i=1}^{N} \Big(-\sum_{j=1}^{P} a_{i,j}\lambda_j + Y_i \log \sum_{j=1}^{P} a_{i,j}\lambda_j - \cancel{Y_i!} \Big) \quad \text{CONSTANT}$$

$$= \sum_{i=1}^{N} \Big(-\underbrace{\sum_{j=1}^{P} a_{i,j}\lambda_j}_{\text{PROJECTION}} + Y_i \log \underbrace{\sum_{j=1}^{P} a_{i,j}\lambda_j}_{\text{PROJECTION}} \Big)$$

SOLVED ITERATIVELY WITH EXPECTATION MAXIMIZATION ITERATION (ML-EM)

$$\lambda_{n+1,j} = \lambda_{n,j} \underbrace{\left(\frac{1}{\sum_{i=1}^{N} a_{ij}} \right)}_{\substack{\text{PROJECT "1"} \\ \text{SENSITIVITY} \\ \text{CORRECTION}}} \underbrace{\sum_{i=1}^{N} a_{i,j} \frac{\overbrace{Y_i}^{\text{DATA}}}{\underbrace{\sum_{j'}^{P} a_{i,j'} \lambda_{n,j'}}_{\substack{\text{PROJECTED} \\ \text{ESTIMATE}}}}}_{\text{BACKPROJECT RATIO}}$$

INITIALIZE WITH $\underline{\lambda}_0 = 1$

PROPORTIONATELY INCREASES / DECREASES VOXEL VALUES

PROPERTIES

COST FUNCTION INCREASES MONOTONICALLY

CONVERGES TO MAX LOG LIKELYHOOD

NON-NEGATIVE

PROBLEMS

CAN OVERFIT DATA

EACH STEP IS FULL 3D BACKPROJECTION

ORDERED SUBSET EXPECTATION MAXIMIZATION (OSEM)

SPLIT DATA INTO S SUBSETS

SETS OF PARALLEL PROJECTIONS

ITERATE OVER ONE SUBSET WITH ML-EM

CYCLE THROUGH SUBSETS

ACCELERATES CONVERGENCE BY ~ S

MANY EXTENSIONS, VARIATIONS