

PARALLEL MRI

COMPUTING GRAPPA WEIGHTS

SPIRiT

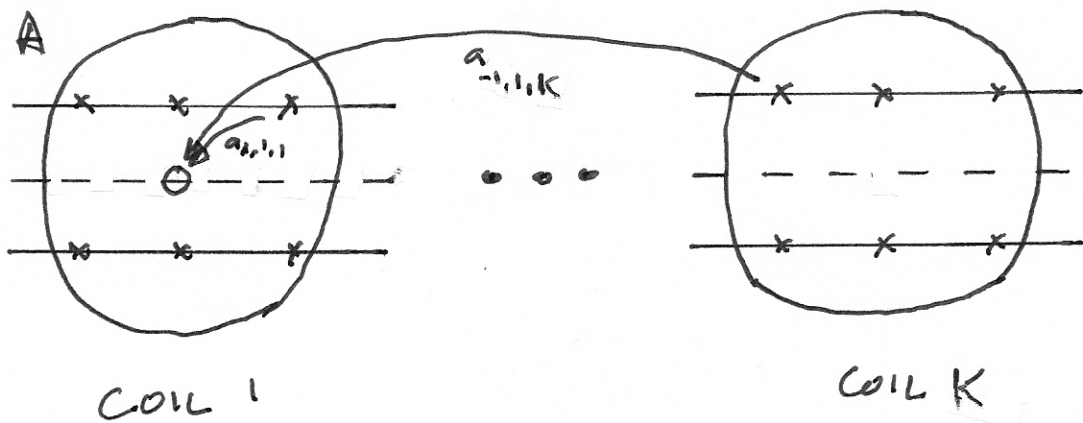
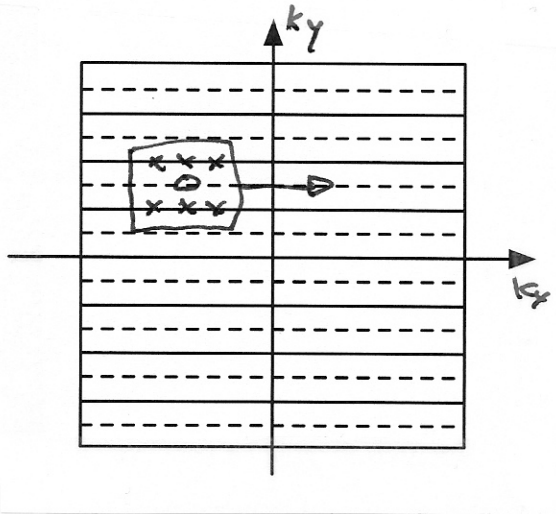
COIL COMPRESSION

ASSIGNMENT

READ LUSTIG SPIRiT PAPER

# G-RAPPA

FILL IN MISSING K-SPACE  
DATA FROM NEIGHBORING  
SAMPLES

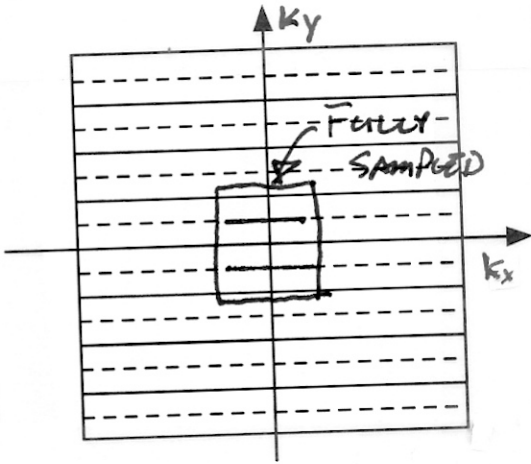


MISSING DATA IS LINEAR COMBINATION OF ALL  
NEIGHBORHOOD POINTS, FROM ALL COILS

$$\hat{M}_k(k_x, k_y) = \sum_{i,j,k} a_{i,j,k} M_k(k_x + i\Delta k, k_y + j\Delta k)$$

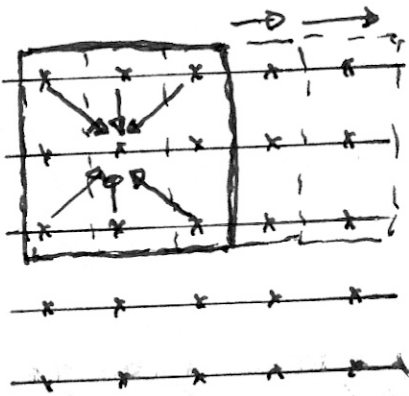
HOW DO WE FIND  $a_{i,j,k}$ ?

# AUTO CALIBRATION



ASSUME THERE IS A REGION WHERE WE ARE FULLY SAMPLED  
 WE HAVE SAMPLES OF WHAT THE GRAPPA SYNTHESIS EQUATION SHOULD PRODUCE  
 INVERT THIS TO SOLVE FOR GRAPPA WEIGHTS

CALIBRATION AREA HAS TO BE LARGER THAN THE GRAPPA KERNEL



EACH SHIFT OF KERNEL GIVES ANOTHER EQUATION  
 HERE 3x3 KERNEL, 5x5 CALIBRATION AREA GIVES 3x3=9 EQUATIONS

WRITE AS A MATRIX EQUATION

$$\underbrace{M}_{\text{CALIBRATION DATA}}_{K,C} = \underbrace{M}_A \underbrace{q}_K$$

GRAPPA COEFFICIENTS  
NEIGHBOURHOOD SAMPLES  
ALL COILS

GRAPPA WEIGHTS ARE

$$\underline{a}_k = \left( M_A^* M_A + \lambda I \right)^{-1} M_A^* \underline{m}_{k,c}$$

WHERE  $\lambda$  IS AN OPTIONAL REGULARIZATION PARAMETER.

REPEAT FOR EACH COIL.

### GRAPPA ALGORITHM

COMPUTE GRAPPA WEIGHTS FROM CALIBRATION REGION

COMPUTE MISSING K-SPACE DATA USING THE GRAPPA WEIGHTS

RECONSTRUCT INDIVIDUAL COIL IMAGES

COMBINE COIL IMAGES

### GRAPPA ISSUES

CALIBRATION REGION SIZE

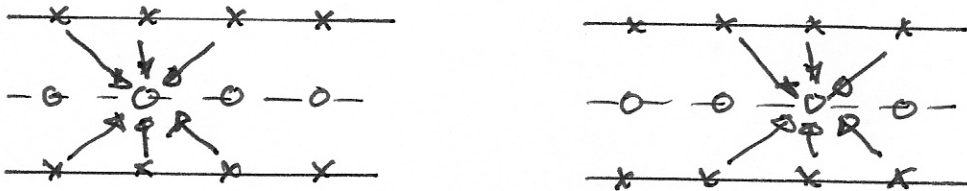
GRAPPA KERNEL SIZE

SAMPLE GEOMETRY DEPENDENCE

# SAMPLE GEOMETRY DEPENDENCE

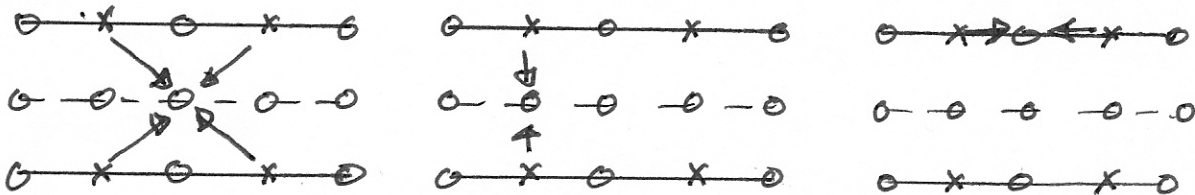
GRAPPA WEIGHTS DEPENDS ON WHICH SAMPLES ARE ACQUIRED, AND WHICH ARE ESTIMATED

SIMPLE FOR SKIPPED LINES



SAME WEIGHTS WORK EVERYWHERE

HARDER FOR ZXZ ACCELERATION



EACH GEOMETRY HAS ITS OWN SET OF WEIGHTS

RAPIDLY GETS CONFUSING

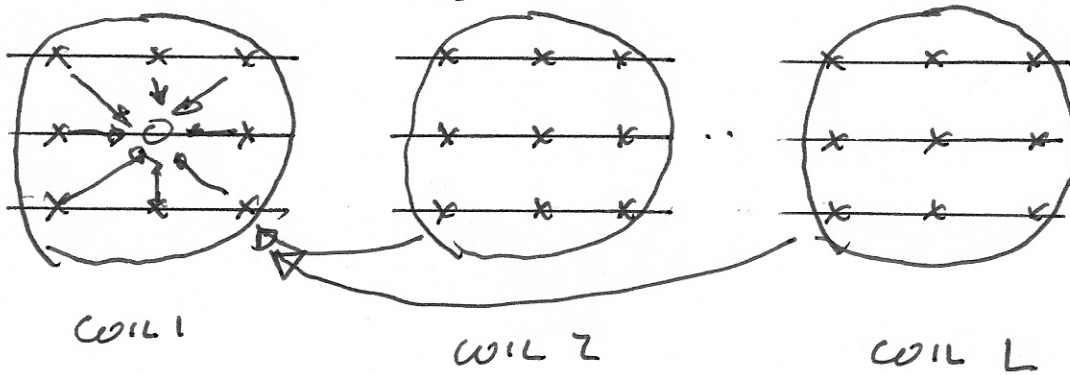
FOR IRREGULAR SAMPLING (NON-CARTESIAN, RANDOM)

BECOMES UNMANAGEABLE!

# ALTERNATIVE PERSPECTIVE

6

ASSUME I KNOW THE FINAL RECONSTRUCTION  
AND I TRY TO ESTIMATE A SAMPLE FROM  
ITS NEIGHBORS



I SHOULD GET MY ORIGINAL RECONSTRUCTION  
THE FULL GRAPPA OPERATOR SHOULD DO NOTHING  
IE I APPLY IT TO THE CORRECT RECONSTRUCTION!  
THE FINAL RECONSTRUCTION SHOULD BE CONSISTENT  
WITH THE FULL GRAPPA OPERATOR.

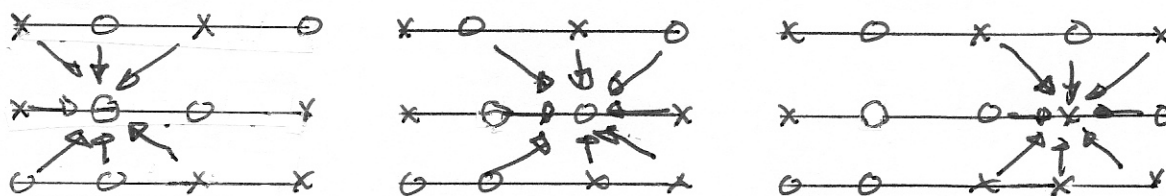
## SPIRiT BASIC IDEA

- 1) CALIBRATE ON ENTIRE NEIGHBORHOOD, KNOWN OR UNKNOWN
- 2) FILL IN KNOWN DATA, ZERO OTHERWISE
- 3) APPLY KERNEL TO ESTIMATE ALL DATA
- 4) RESTORE KNOWN DATA
- 5) ITERATE 3+4 UNTIL DATA STOPS CHANGING

THIS IS A 'PROJECTION ONTO CONVEX SETS'  
OR POCS TYPE ALGORITHM

ALTERNATELY ENFORCES TWO CONSTRAINTS  
CONSISTENCY WITH ACQUIRED DATA  
CONSISTENCY WITH KERNEL

SAME WEIGHTS FOR ANY GEOMETRY



ZERO VALUES SLOWLY FILL IN

ADVANTAGES

- ONLY ONE SET OF WEIGHTS
- WORKS FOR ANY SAMPLE GEOMETRY
- EFFECTIVELY A LARGER KERNEL

DISADVANTAGE

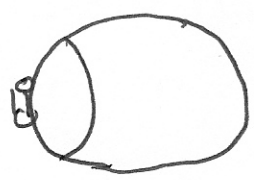
ITERATIVE RECON

# EIGEN COILS AND COIL COMPRESSION

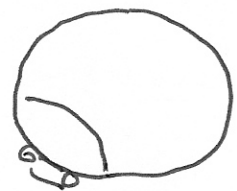
## ARRAY COIL SENSITIVITIES



COIL 1

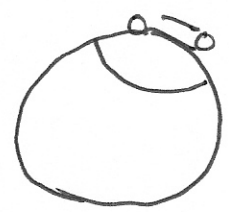


COIL 2



COIL 3

...



COIL L

EACH COIL SEES A LOCAL REGION

NOT CLEAR HOW MUCH ACCELERATION IS POSSIBLE

g-FACTOR HITS A WALL AT 3-4 IN 1D, WHY?

WHAT IS THE FUNDAMENTAL DIMENSIONALITY OF RF ENCODING

## EIGEN COILS

MAKE A MATRIX OF VECTORIZED SENSITIVITY MAPS

$$C = \begin{pmatrix} \vdots & \vdots & \vdots \\ c_1(x) & c_2(x) & \dots & c_L(x) \\ \vdots & \vdots & \vdots \end{pmatrix}$$

THE MATRIX  $C^*C$  SHOWS THE CORRELATION BETWEEN CHANNELS



COMPUTE THE EIGEN DECOMPOSITION OF  $C^*C$

$$C^*C = BDB^*$$

WHERE  $B$  IS A UNITARY MATRIX OF EIGENVECTORS AND

$$D = \begin{pmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \lambda_L \end{pmatrix}$$

IS A DIAGONAL MATRIX OF EIGENVALUES

THEN

$$B^* \underbrace{C^*C}_{C'} B = D$$

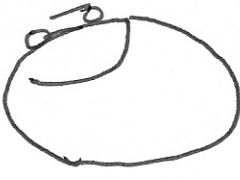
LET

$$\underbrace{C'}_{\text{EIGEN COILS}} = \underbrace{C}_{\text{COILS}} \underbrace{B}_{\text{UNITARY ROTATION}}$$

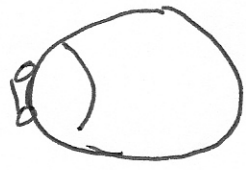
THE  $\lambda_i$  TELL YOU HOW MUCH ENERGY IS IN EACH EIGEN COIL CHANNEL

THESE DROP OFF RAPIDLY. THIS TELLS YOU HOW MANY INDEPENDENT CHANNELS YOU HAVE WHERE THAT ADDITIONAL INFORMATION IS

ORIGINAL CHANNELS



COIL 1



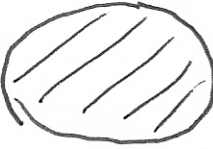
COIL 2



COIL 3

...

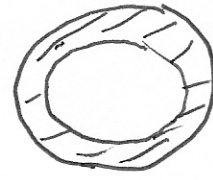
EIGEN CHANNELS



UNIFORM



1<sup>ST</sup> MODE



2<sup>ND</sup> MODE

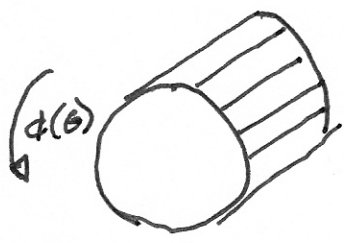
CHANNELS GET:

CLOSER TO EDGE

LOWER MAXIMUM

INCREASING NUMBER OF CYCLES OF PHASE

LOOKS LIKE THE MODES OF A BIRISCAGE RF COIL



UNIFORM - DRIVE IN PHASE

1<sup>ST</sup> MODE - ONE CYCLE OF PHASE

2<sup>ND</sup> MODE - TWO CYCLES OF PHASE

⋮

⋮

YOU CAN DO PARALLEL IMAGING THIS WAY!

EIGEN COILS SHOW YOU SOMETHING FUNDAMENTAL ABOUT THE ELECTROMAGNETICS

## COIL COMPRESSION

(11)

USE THE EIGENCOIL BASIS TO REDUCE THE SIZE OF YOUR PARALLEL IMAGING RECON

IF  $M$  IS A MATRIX OF THE VECTORIZED ACQUIRED DATA, COMPUTE

$$M' = M\beta$$

THIS IS THE DATA ROTATED INTO THE EIGEN COIL SPACE.

ONLY KEEP THE COLUMNS OF  $M'$  THAT HAVE SIGNIFICANT EIGEN COILS

RECONSTRUCT USING EIGENCOILS  $C'$ .